CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
20MCA261	OPERATIONS RESEARCH	ELECTIVE	3	1	0	4

Preamble: This course introduces the concepts of linear programming problems. The topics treated in this course have applications in real life problems.

Prerequisite: Nil

Course Outcomes: After completion of the course the student will be able to

CO No.	Course Outcome (CO)	Bloom's Category Level
CO 1	Solve different types of Linear Programming Problems.	Level 3: Apply
CO 2	Apply the concept of linear programming problems in real life.	Level 3: Apply
CO 3	Solve different decision-making problems using optimization techniques.	Level 3: Apply
CO 4	Use PERT and CPM to analyse project network management.	Level 3: Apply
CO 5	Identify suitable queuing model and solve queuing problems.	Level 3: Apply

Mapping of Course Outcomes with Program Outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	3	1	-	-	-	2	-	-	-	-	-
CO 2	3	3	3	-	-	-	2	-	-	-	-	-
CO 3	3	3	3	-	-	-	2	-	-	-	-	-
CO 4	3	3	1	1	-	-	2	2	-	-	-	-
CO 5	3	3	3	-	-	-	2	-	-	-	-	-

3/2/1: High/Medium/Low

Assessment Pattern

Bloom's Category Levels	Continuous Assessment Tests		End Semester Examination
	1	2	
Level 1: Remember	10	10	10
Level 2: Understand	20	20	20
Level 3: Apply	20	20	30
Level 4: Analyse			
Level 5: Evaluate			
Level 6: Create			

Mark Distribution

Total Marks	Continuous Internal Evaluation (CIE)	End Semester Examination (ESE)	ESE Duration
100	40	60	3 hours

Continuous Internal Evaluation Pattern:

Attendance : 8 marks
Continuous Assessment Test (2 numbers) : 20 marks
Assignment/Quiz/Course project : 12 marks

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 compulsory short answer questions, 2 from each module. Each question carries 3 marks. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 6 marks

Sample Course Level Assessment Questions

Course Outcome 1 (CO 1):

1. Define slack variable, surplus variable and optimal basic feasible solution.
2. Obtain all basic feasible solution of the set of equations:
   a) $2x_1 + 3x_2 + 4x_3 + x_4 = 2$
   b) $x_1 + x_2 + 7x_3 + x_4 = 4$
3. Solve by Big M method

Maximise $Z = 6x_1 - 3x_2 + 2x_3$
Subject to $2x_1 + x_2 + x_3 \leq 16$
$3x_1 + 2x_2 + x_3 \leq 18$

$$x_1 - 2x_2 \geq 8$$

$$x_1,\ x_2,\ x_3 \geq 0$$

Course Outcome 2 (CO 2):

1. Construct the dual of

$$\begin{aligned} &\text{Maximise} && Z = 3x_1 + 17x_2 + 9x_3 \\ &\text{Subject to} && x_1 - x_2 + x_3 \geq 3 \\ & && -3x_1 + 2x_2 \leq 1 \\ & && x_1,\ x_2,\ x_3 \geq 0 \end{aligned}$$

2. Prove that the dual of the dual is the primal
3. Solve using the principle of duality

$$\begin{aligned} &\text{Minimise} && Z = 3x_1 + 5x_2 \\ &\text{Subject to} && 2x_1 + 8x_2 \geq 40 \\ & && 3x_1 + 4x_2 \geq 50 \\ & && x_1,\ x_2 \geq 0 \end{aligned}$$

Course Outcome 3 (CO 3):

1. Explain North West Corner method
2. Solve the following transportation problem

	1	2	3	Supply
1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	34

3. Solve the assignment problem

	I	II	III	IV
A	16	10	14	11
B	14	11	15	15
C	15	15	13	12
D	13	12	14	15

Course Outcome 4 (CO 4):

1. Explain critical path analysis.
2. A project consists of series of tasks labelled A, B, …, H, I with the following relationships (W < X, Y means X and Y cannot start until W is completed; X, Y < W means W cannot start until both X and Y are completed). With this notation construct the network diagram having the following constraints:

   A < D, E; B, D < F; C < G; B, G< H; F, G < I.

   Find also the minimum time of completion of the project, when the time (in days) of completion of each task is as follows:

Task :	A	B	C	D	E	F	G	H	I
Time :	23	8	20	16	24	18	19	4	10

3. A project consists of eight activities with the following relevant information.

Activity	Immediate predecessor	Estimated duration (days)		
		Optimistic	Most likely	Pessimistic
A	--	1	1	7
B	--	1	4	7
C	--	2	2	8
D	A	1	1	1
E	B	2	5	14
F	C	2	5	8
G	D, E	3	6	15
H	F, G	1	2	3

(i) Draw the PERT network and find out the expected project completion time.
(ii) What duration will have 95% confidence for project completion?
(iii) If the average duration for activity F increases to 14 days, what will be its effects on the expected project completion time which will have 95% confidence?

(For standard normal Z = 1.645, area under the standard normal curve from 0 to Z is 0.45)

Course Outcome 5 (CO 5):

1. Explain Birth-death process.
2. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the

service time distribution is also exponential with an average 36 minutes. Calculate the following:

i. The mean queue size (line length), and
ii. The probability that the queue size exceeds 10.
iii. If the input of trains increases to an average 33 per day, what will be the change in (i) and (ii)?

3. At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady-state probabilities for the various number of trains in the system. also find the average waiting time of a new train coming into the yard

Model Question Paper
Course Code: 20MCA261
Course name: Operations Research

Max. Marks: 60 Duration: 3hrs

Part A
Answer all questions, each carries 3 marks (10×3 = 30)

1. Write down the basic structure of a linear programming problem in the mathematical form.
2. Define slack and surplus variables in LPP.
3. State the fundamental theorem of duality.
4. Write the dual of the following

$$Max\ Z = x_1 - x_2 + 3x_3$$
$$subject\ to\ x_1 + x_2 + x_3 \leq 10$$
$$2x_1 - x_3 \leq 2$$
$$2x_1 - 2x_2 + 3x_3 \leq 6$$
$$x_1, x_2, x_3 \geq 0$$

5. Obtain the IBFS using north west corner method

	D_1	D_2	D_3	D_4	Supply
O_1	2	4	3	6	20
O_2	7	3	8	2	10
O_3	2	2	9	11	15
Demand	15	15	8	7	

6. Describe the Matrix Minima method.
7. What is queue discipline?
8. Explain single serve Poisson queuing model with infinite capacity.
9. Activities P, Q and R instantly follow activity M, and their current starting times are 12, 19 and 10. So, what is the latest finishing time for activity M?
10. What is the difference between PERT and CPM.

Part B

Answer all questions, each carries 6 marks (5×6 = 30)

11. Solve the following problem by Simplex method

$$Max\ Z = 5x_1 + 3x_2$$
$$subject\ to\ 4x_1 - x_2 \leq 10$$
$$2x_1 + 2x_2 \leq 50$$
$$x_1, x_2 \geq 0$$

or

12. Solve by Big-M method

$$Max\ Z = 6x_1 - 3x_2 + 2x_3$$
$$subject\ to\ 2x_1 + x_2 + x_3 \leq 16$$
$$3x_1 + 2x_2 + x_3 \leq 18$$
$$x_2 - 2x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0$$

13. Prove that the dual of a dual is the primal.

or

14. Solve the following by using the dual principle

$$Max\ Z = 40x_1 + 35x_2$$
$$subject\ to\ 2x_1 + 3x_2 \leq 60$$
$$4x_1 + 3x_2 \leq 96$$
$$x_1, x_2 \geq 0$$

15. Solve the following Assignment problem

	I	II	III	IV
A	2	3	4	5
B	4	5	6	7
C	7	8	9	8
D	3	5	8	9

or

16. Solve the following transportation problem

	D_1	D_2	D_3	D_4	Supply
O_1	5	2	4	3	22
O_2	4	5	1	6	15
O_3	4	6	7	5	8
Demand	7	12	17	9	

17. Explain critical path analysis.

or

18. A project consists of eight activities with the following relevant information.

Activity	Immediate predecessor	Estimated duration (days)		
		Optimistic	Most likely	Pessimistic
A	--	1	1	7
B	--	1	4	7
C	--	2	2	8
D	A	1	1	1
E	B	2	5	14
F	C	2	5	8
G	D, E	3	6	15
H	F, G	1	2	3

(iv) Draw the PERT network and find out the expected project completion time.
(v) What duration will have 95% confidence for project completion?
(vi) If the average duration for activity F increases to 14 days, what will be its effects on the expected project completion time which will have 95% confidence?

(For standard normal Z = 1.645, area under the standard normal curve from 0 to Z is 0.45)

19. Explain birth-death process.

or

20. At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady-state probabilities for the various number of trains in the system. also find the average waiting time of a new train coming into the yard.

Syllabus

Module 1: (9 Hours)
Linear programming problem- Slack and surplus variable- Standard form- Solution of Linear programming problem- Basic solution- Basic feasible solution- Degenerate- and Non-degenerate solutions- Optimal solution- Solution by simplex method- Artificial variables- Big- M method.
Module 2: (9 Hours)
Duality in Linear Programming Problem- Statement of duality theorem- Statement of complementary slackness theorem. The primal- Duality solutions using simplex method- Revised simplex method
Module 3: (9 Hours)
Transportation problem- Solution of Transportation problem- Finding an initial basic feasible solution- North West Corner method- Matrix minima method- Vogel's Approximation method- Test for Optimality- Modi method- Unbalanced Transportation problem- Maximisation in Transportation problem. Assignment problem- Optimal solution- Hungarian method of assignment- Maximization in assignment problem.
Module 4: (9 Hours)
Network analysis- Project scheduling- Construction of project networks- Critical path method (CPM)- Identification of critical path using CPM- Estimation of Floats- Total float- Independent float- Project Evaluation and Review Technique (PERT) - Computation of expected completion times by PERT.
Module 5: (9 Hours)
Queuing theory- Elements of Queuing System- Kendall's notation- Operating characteristics- Poisson process- Exponential distribution- Mean and variance- Birth and Death process. Queuing models based on Poisson process- Single server models with finite and infinite capacity- Multi server model with finite and infinite capacity.

Note:

- Programming Assignments using Python and appropriate Case Studies may be given at the end of each module.
- Linear Programming Problems in module 1 and module 2 and Transportation problems in module 3 can be solved using Python library PuLP. Using Numpy, PERT/CPM problems in module 4 can be solved.

Text Book

1. KantiSwarup, P.K. Gupta and Man Mohan, Operation Research, Sultan Chand (2010)

Reference Books

1. Hamdy A Taha, Operations Research- an introduction, Eighth Edition, Prentice Hall of India.
2. Ravindran, Philips and Solberg, Wiley, Operation Research, Second edition (2007)

Web References

1. https://pypi.org/project/PuLP/
2. https://numpy.org/

Course Contents and Lecture Schedule

Sl. No.	Topic	No. of Lectures
1	**Module 1**	**9 Hours**
1.1	Linear programming problem- Slack and surplus variable- Standard form	1
1.2	Solution of Linear programming problem- Basic solution- Basic feasible solution- Degenerate- and Non-degenerate solutions- Optimal solution	2
1.3	Solution by simplex method	3
1.4	Artificial variables- Big- M method	3
2	**Module 2**	**9 Hours**
2.1	Duality in Linear Programming Problem	1
2.2	Statement of duality theorem- Statement of complementary slackness theorem	2
2.3	The primal- Duality solutions using simplex method	3
2.4	Revised simplex method	3
3	**Module 3**	**9 Hours**
3.1	Transportation problem- Solution of Transportation problem- Finding an initial basic feasible solution- North West Corner method	2
3.2	Matrix minima method- Vogel’s Approximation method	1
3.3	Test for Optimality- Modi method- Unbalanced Transportation problem- Maximisation in Transportation problem	3
3.4	Assignment problem- Optimal solution- Hungarian method of assignment- Maximization in assignment problem	3
4	**Module 4**	**9 Hours**
4.1	Network analysis- Project scheduling- Construction of project networks	1
4.2	Critical path method (CPM)- Identification of critical path using CPM	2
4.3	Estimation of Floats- Total float- Independent float	3
4.4	Project Evaluation and Review Technique (PERT)	2
4.5	Computation of expected completion times by PERT	1

5	**Module 5**	**9 Hours**
5.1	Queuing theory- Elements of Queuing System- Kendall’s notation- Operating characteristics- Poisson process	1
5.2	Exponential distribution- Mean and variance- Birth and Death process	2
5.3	Queuing models based on Poisson process	3
5.4	Single server models with finite and infinite capacity	1
5.5	Multi server model with finite and infinite capacity	2