32. TRUE DISCOUNT

IMPORTANT CONCEPTS

Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to Rs. 156 in 4 years. So, the payment of Rs. 100 now will clear off the debt of Rs. 156 due 4 years hence. We say that :

Sum due = Rs. 156 due 4 years hence;

Present Worth (P.W.) = Rs. 100;

True Discount (T.D.) = Rs. (156 – 100) = Rs. 56 = (Sum due) – (P.W.).

We define : ***T.D. = Interest on P.W.***

Amount = (P.W.) + (T.D.).

Interest is reckoned on P.W. and true discount is reckoned on the amount.

IMPORTANT FORMULAE

Let rate = R% per annum and Time = T years. Then,

1. $\text{P.W.} = \dfrac{100 \times \text{Amount}}{100 + (\text{R} \times \text{T})} = \dfrac{100 \times \text{T.D.}}{\text{R} \times \text{T}}$.

2. $\text{T.D.} = \dfrac{(\text{P.W.}) \times \text{R} \times \text{T}}{100} = \dfrac{\text{Amount} \times \text{R} \times \text{T}}{100 + (\text{R} \times \text{T})}$.

3. $\text{Sum} = \dfrac{(\text{S.I.}) \times (\text{T.D.})}{(\text{S.I.}) - (\text{T.D.})}$.

4. (S.I.) – (T.D.) = S.I. on T.D.

5. When the sum is put at compound interest, then $\text{P.W.} = \dfrac{\text{Amount}}{\left(1 + \dfrac{\text{R}}{100}\right)^{\text{T}}}$

SOLVED EXAMPLES

Ex. 1. ***Find the present worth of Rs. 930 due 3 years hence at 8% per annum. Also find the discount.***

Sol. $\text{P.W.} = \dfrac{100 \times \text{Amount}}{100 + (\text{R} \times \text{T})} = \text{Rs.}\left[\dfrac{100 \times 930}{100 + (8 \times 3)}\right] = \text{Rs.}\left(\dfrac{100 \times 930}{124}\right) = \text{Rs. } 750.$

T.D. = (Amount) – (P.W.) = Rs. (930 – 750) = Rs. 180.

Ex. 2. ***The true discount on a bill due 9 months hence at 12% per annum is Rs. 540. Find the amount of the bill and its present worth.***

Sol. Let amount be Rs. x. Then,

$$\frac{x \times \text{R} \times \text{T}}{100 + (\text{R} \times \text{T})} = \text{T.D.} \Rightarrow \frac{x \times 12 \times \frac{3}{4}}{100 + \left(12 \times \frac{3}{4}\right)} = 540 \Rightarrow x = \left(\frac{540 \times 109}{9}\right) = \text{Rs. } 6540$$

$\therefore$ Amount = Rs. 6540.

P.W. = Rs. (6540 – 540) = Rs. 6000.

33. BANKER'S DISCOUNT

IMPORTANT CONCEPTS

Banker's Discount : Suppose a merchant A buys goods worth, say Rs. 10,000 from another merchant B at a credit of say 5 months. Then, B prepares a bill, called the bill of exchange. A signs this bill and allows B to withdraw the amount from his bank account after exactly 5 months.

The date exactly after 5 months is called *nominally due date*. Three days (known as *grace days*) are added to it to get a date, known as *legally due date*.

Suppose B wants to have the money before the legally due date. Then he can have the money from the banker or a broker, who deducts S.I. on the face value (*i.e.*, Rs. 10,000 in this case) for the period from the date on which the bill was discounted (*i.e.*, paid by the banker) and the legally due date. This amount is known as ***Banker's Discount* (B.D.)**

Thus, **B.D.** *is the S.I. on the face value for the period from the date on which the bill was discounted and the legally due date.*

Banker's Gain (B.G.) = (B.D.) – (T.D.) *for the unexpired time.*

Note : When the date of the bill is not given, grace days are not to be added.

IMPORTANT FORMULAE

1. B.D. = S.I. on bill for unexpired time.
2. B.G. = (B.D.) – (T.D.) = S.I. on T.D. = $\frac{(\text{T.D.})^2}{\text{P.W.}}$.
3. T.D. = $\sqrt{\text{P.W.} \times \text{B.G.}}$
4. B.D. = $\left(\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100}\right)$.
5. T.D. = $\left[\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + (\text{Rate} \times \text{Time})}\right]$.
6. Amount = $\left(\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}\right)$.
7. T.D. = $\left(\frac{\text{B.G.} \times 100}{\text{Rate} \times \text{Time}}\right)$

SOLVED EXAMPLES

Ex. 1. ***A bill for Rs. 6000 is drawn on July 14 at 5 months. It is discounted on 5th October at 10%. Find the banker's discount, true discount, banker's gain and the money that the holder of the bill receives.***

Sol. Face value of the bill = Rs. 6000.

Date on which the bill was drawn = July 14 at 5 months.

Nominally due date = December 14. Legally due date = December 17.

Date on which the bill was discounted = October 5.

Unexpired time : Oct. Nov. Dec.

$$26 + 30 + 17 = 73 \text{ days} = \frac{1}{5} \text{ year.}$$

$\therefore$ B.D. = S.I. on Rs. 6000 for $\frac{1}{5}$ year = Rs. $\left(6000 \times 10 \times \frac{1}{5} \times \frac{1}{100}\right)$ = Rs. 120.

T.D. = Rs. $\left[\frac{6000 \times 10 \times \frac{1}{5}}{100 + \left(10 \times \frac{1}{5}\right)}\right]$ = Rs. $\left(\frac{12000}{102}\right)$ = Rs. 117.64.

$\therefore$ B.G. = (B.D.) − (T.D.) = Rs. (120 − 117.64) = Rs. 2.36.

Money received by the holder of the bill = Rs. (6000 − 120) = Rs. 5880.

Ex. 2. ***If the true discount on a certain sum due 6 months hence at 15% is Rs. 120, what is the banker's discount on the same sum for the same time and at the same rate?***

Sol. B.G. = S.I. on T.D. = Rs. $\left(120 \times 15 \times \frac{1}{2} \times \frac{1}{100}\right)$ = Rs 9.

$\therefore$ (B.D.) − (T.D.) = Rs. 9.

$\therefore$ B.D. = Rs. (120 + 9) = Rs. 129.

Ex. 3. ***The banker's discount on Rs. 1800 at 12% per annum is equal to the true discount on Rs. 1872 for the same time at the same rate. Find the time.***

Sol. S.I. on Rs. 1800 = T.D. on Rs. 1872.

$\therefore$ P.W. of Rs. 1872 is Rs. 1800.

$\therefore$ Rs. 72 is S.I. on Rs. 1800 at 12%.

$\therefore$ Time = $\left(\frac{100 \times 72}{12 \times 1800}\right)$ year = $\frac{1}{3}$ year = 4 months.

Ex. 4. ***The banker's discount and the true discount on a sum of money due 8 months hence are Rs. 120 and Rs. 110 respectively. Find the sum and the rate percent.***

Sol. Sum = $\left(\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}}\right)$ = Rs. $\left(\frac{120 \times 110}{120 - 110}\right)$ = Rs. 1320.

Since B.D. is S.I. on sum due, so S.I. on Rs. 1320 for 8 months is Rs. 120.

$\therefore$ Rate = $\left(\frac{100 \times 120}{1320 \times \frac{2}{3}}\right)\% = 13\frac{7}{11}\%$.

Ex. 5. ***The present worth of a bill due sometime hence is Rs. 1100 and the true discount on the bill is Rs. 110. Find the banker's discount and the banker's gain.***

Sol. T.D. = $\sqrt{\text{P.W.} \times \text{B.G.}}$

$\therefore$ B.G. = $\frac{(\text{T.D.})^2}{\text{P.W.}}$ = Rs. $\left(\frac{110 \times 110}{1100}\right)$ = Rs. 11.

$\therefore$ B.D. = (T.D. + B.G.) = Rs. (110 + 11) = Rs. 121.

Ex. 6. ***The banker's discount on Rs. 1650 due a certain time hence is Rs. 165. Find the true discount and the banker's gain.***

Sol. Sum = $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{\text{B.D.} \times \text{T.D.}}{\text{B.G.}}$.

$\therefore$ $\frac{\text{T.D.}}{\text{B.G.}} = \frac{\text{Sum}}{\text{B.D.}} = \frac{1650}{165} = \frac{10}{1}$.

Thus, if B.G. is Re 1, T.D. = Rs. 10.

If B.D. is Rs. 11, T.D. = Rs. 10. If B.D. is Rs. 165, T.D. = Rs. $\left(\frac{10}{11} \times 165\right)$ = Rs. 150.

And, B.G. = Rs. (165 − 150) = Rs. 15.

34. HEIGHTS AND DISTANCES

IMPORTANT FACTS AND FORMULAE

1. We already know that :
   In a rt. angled ΔOAB, where $\angle BOA = \theta$,

   (*i*) $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{AB}{OB}$;

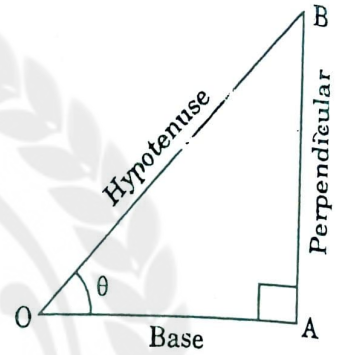
   (*ii*) $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{OA}{OB}$;

   (*iii*) $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{AB}{OA}$;

   (*iv*) $\text{cosec}\,\theta = \dfrac{1}{\sin\theta} = \dfrac{OB}{AB}$;

   (*v*) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{OB}{OA}$;

   (*vi*) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{OA}{AB}$.

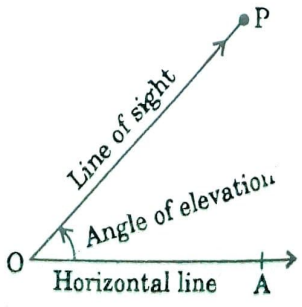


2. **Trigonometrical Identities :**
   (*i*) $\sin^2\theta + \cos^2\theta = 1$. (*ii*) $1 + \tan^2\theta = \sec^2\theta$. (*iii*) $1 + \cot^2\theta = \text{cosec}^2\theta$.

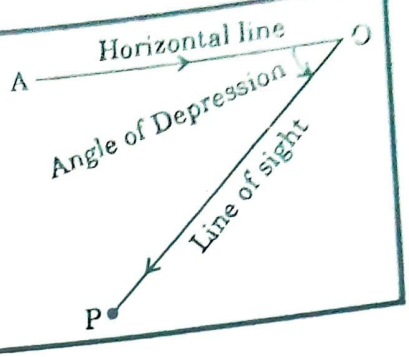
3. **Values of T-ratios :**

θ	$0°$	$(\pi/6)$ $30°$	$(\pi/4)$ $45°$	$(\pi/3)$ $60°$	$(\pi/2)$ $90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

4. **Angle of Elevation :** Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.
   $\therefore$ Angle of elevation of P from O = $\angle AOP$.



5. **Angle of Depression** : Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.



SOLVED EXAMPLES

Ex. 1. *If the height of a pole is $2\sqrt{3}$ metres and the length of its shadow is 2 metres, find the angle of elevation of the sun.*

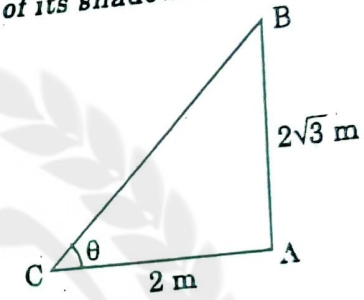
Sol. Let AB be the pole and AC be its shadow.

Let angle of elevation, $\angle ACB = \theta$.

Then, AB = $2\sqrt{3}$ m, AC = 2 m.

$$\tan\theta = \frac{AB}{AC} = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = 60°$$

So, the angle of elevation is 60°.



Ex. 2. *A ladder leaning against a wall makes an angle of 60° with the ground. If the length of the ladder is 19 m, find the distance of the foot of the ladder from the wall.*

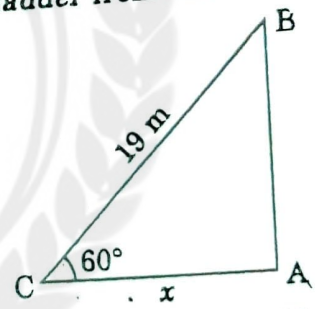
Sol. Let AB be the wall and BC be the ladder.

Then, $\angle ACB = 60°$ and BC = 19 m.

Let AC = x metres

$$\frac{AC}{BC} = \cos 60° \Rightarrow \frac{x}{19} = \frac{1}{2} \Rightarrow x = \frac{19}{2} = 9.5.$$

$\therefore$ Distance of the foot of the ladder from the wall = 9.5 m.



Ex. 3. *The angle of elevation of the top of a tower at a point on the ground is 30°. On walking 24 m towards the tower, the angle of elevation becomes 60°. Find the height of the tower.*

Sol. Let AB be the tower and C and D be the points of observation. Then,

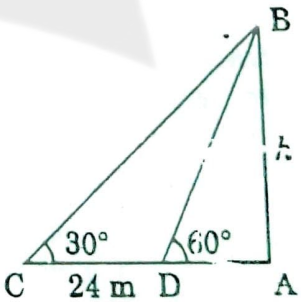
$$\frac{AB}{AD} = \tan 60° = \sqrt{3} \Rightarrow AD = \frac{AB}{\sqrt{3}} = \frac{h}{\sqrt{3}}.$$

$$\frac{AB}{AC} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \times \sqrt{3} = h\sqrt{3}.$$

$$CD = (AC - AD) = \left(h\sqrt{3} - \frac{h}{\sqrt{3}}\right).$$

$\therefore$ $h\sqrt{3} - \frac{h}{\sqrt{3}} = 24 \Rightarrow h = 12\sqrt{3} = (12 \times 1.73) = 20.76.$

Hence, the height of the tower is 20.76 m.



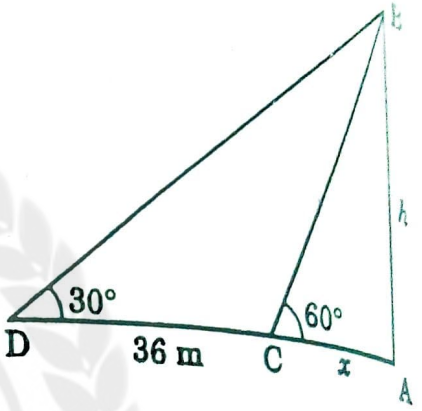
Ex. 4. *A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60° When he retires 36 m from the bank, he finds the angle to be 30°. Find the breadth of the river.*

Sol. Let AB be the tree and AC be the river. Let C and D be the two positions of the man. Then,

$\angle ACB = 60°$, $\angle ADB = 30°$ and $CD = 36$ m.

Let $AB = h$ metres and $AC = x$ metres.

Then, $AD = (36 + x)$ metres.



$$\frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{36+x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{36+x}{\sqrt{3}} \qquad ...(i)$$

$$\frac{AB}{AC} = \tan 60° = \sqrt{3} \Rightarrow \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow \quad h = \sqrt{3}\,x \qquad ...(ii)$$

From (i) and (ii), we get : $\frac{36+x}{\sqrt{3}} = \sqrt{3}\,x \Rightarrow x = 18$ m.

So, the breadth of the river = 18 m.

Ex. 5. ***A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60°. Find the time taken by the boat to reach the shore from this position.***

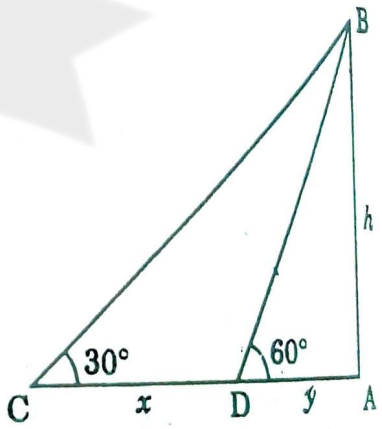
Sol. Let AB be the tower and C and D be the two positions of the boat.

Let $AB = h$, $CD = x$ and $AD = y$.

$$\frac{h}{y} = \tan 60° = \sqrt{3} \qquad \Rightarrow \quad y = \frac{h}{\sqrt{3}}.$$

$$\frac{h}{x+y} = \tan 30° = \frac{1}{\sqrt{3}} \qquad \Rightarrow \quad x + y = \sqrt{3}\,h.$$

$$\therefore \quad x = (x+y) - y = \left(\sqrt{3}\,h - \frac{h}{\sqrt{3}}\right) = \frac{2h}{\sqrt{3}}.$$



Now, $\frac{2h}{\sqrt{3}}$ is covered in 10 min.

$$\therefore \quad \frac{h}{\sqrt{3}} \text{ will be covered in } \left(10 \times \frac{\sqrt{3}}{2h} \times \frac{h}{\sqrt{3}}\right) = 5 \text{ min.}$$

Hence, required time = 5 minutes.

Ex. 6. ***The angles of elevation of the top of a tower from two points on each bank of a river, just opposite to each other***

35. ODD MAN OUT AND SERIES

EXERCISE 35

Directions : *Find the odd man out :*

1. 3, 5, 7, 12, 17, 19
   (a) 19 (b) 17 (c) 13 (d) 12
2. 10, 14, 16, 18, 21, 24, 26
   (a) 26 (b) 24 (c) 21 (d) 18
3. 3, 5, 9, 11, 14, 17, 21
   (a) 21 (b) 17 (c) 14 (d) 9
4. 1, 4, 9, 16, 23, 25, 36
   (a) 9 (b) 23 (c) 25 (d) 36
5. 6, 9, 15, 21, 24, 28, 30
   (a) 28 (b) 21 (c) 24 (d) 30
6. 41, 43, 47, 53, 61, 71, 73, 81
   (a) 61 (b) 71 (c) 73 (d) 81
7. 16, 25, 36, 72, 144, 196, 225
   (a) 36 (b) 72 (c) 196 (d) 225
8. 10, 25, 45, 54, 60, 75, 80
   (a) 10 (b) 45 (c) 54 (d) 75
9. 1, 4, 9, 16, 20, 36, 49
   (a) 1 (b) 9 (c) 20 (d) 49
10. 8, 27, 64, 100, 125, 216, 343
    (a) 27 (b) 100 (c) 125 (d) 343
11. 1, 5, 14, 30, 50, 55, 91
    (a) 5 (b) 50 (c) 55 (d) 91
12. 385, 462, 572, 396, 427, 671, 264
    (a) 385 (b) 427 (c) 671 (d) 264
13. 835, 734, 642, 751, 853, 981, 532
    (a) 751 (b) 853 (c) 981 (d) 532
14. 331, 482, 551, 263, 383, 242, 111
    (a) 263 (b) 383 (c) 242 (d) 111
15. 2, 5, 10, 17, 26, 37, 50, 64
    (a) 50 (b) 26 (c) 37 (d) 64
16. 19, 28, 39, 52, 67, 84, 102
    (a) 52 (b) 102 (c) 84 (d) 67
17. 253, 136, 352, 460, 324, 631, 244
    (a) 136 (b) 324 (c) 352 (d) 631
18. 2, 5, 10, 50, 500, 5000
    (a) 0 (b) 5 (c) 10 (d) 5000
19. 4, 5, 7, 10, 14, 18, 25, 32
    (a) 7 (b) 14 (c) 18 (d) 33

36. TABULATION

This section comprises of questions in which certain data regarding common disciplines as production over a period of a few years : imports, exports, incomes of employees in a factory, students applying for and qualifying a certain field of study etc. are given in the form of a table. The candidate is required to understand the given information and thereafter answer the given questions on the basis of comparative analysis of the data.

Thus, here the data collected by the investigator are arranged in a systematic form in a table called the *tabular form*. In order to avoid some heads again and again, tables are made consisting of horizontal lines called *rows* and vertical lines called *columns* with distinctive heads, known as *captions*. Units of measurements are given with the captions.

SOLVED EXAMPLES

Ex. 1. The following table gives the sales of batteries manufactured by a company over the years. Study the table and answer the questions that follow.

(S.B.I.P.O. 1998)

NUMBER OF DIFFERENT TYPES OF BATTERIES SOLD BY A COMPANY OVER THE YEARS (NUMBERS IN THOUSANDS)

Year	TYPES OF BATTERIES					
	4AH	7AH	32AH	35AH	55AH	Total
1992	75	144	114	102	108	543
1993	90	126	102	84	126	528
1994	96	114	75	105	135	525
1995	105	90	150	90	75	510
1996	90	75	135	75	90	465
1997	105	60	165	45	120	495
1998	115	85	160	100	145	605

1. The total sales of all the seven years is the maximum for which battery ?
   (a) 4AH (b) 7AH (c) 32AH (d) 35AH (e) 55AH
2. What is the difference in the number of 35AH batteries sold in 1993 and 1997 ?
   (a) 24000 (b) 28000 (c) 35000 (d) 39000 (e) 42000
3. The percentage of 4AH batteries sold to the total number of batteries sold was maximum in the year :
   (a) 1994 (b) 1995 (c) 1996 (d) 1997 (e) 1998
4. In the case of which battery there was a continuous decrease in sales from 1992 to 1997 ?
   (a) 4AH (b) 7AH (c) 32AH (d) 35AH (e) 55AH
5. What was the approximate percentage increase in the sales of 55AH batteries in 1998 compared to that in 1992 ?
   (a) 28% (b) 31% (c) 33% (d) 34% (e) 37%

Sol. 1. (*c*) : The total sales (in thousands) of all the seven years for various batteries are :

For 4AH = 75 + 90 + 96 + 105 + 90 + 105 + 115 = 676

For 7AH = 144 + 126 + 114 + 90 + 75 + 60 + 85 = 694

For 32AH = 114 + 102 + 75 + 150 + 135 + 165 + 160 = 901

For 35 AH = 102 + 84 + 105 + 90 + 75 + 45 + 100 = 601

For 55 AH = 108 + 126 + 135 + 75 + 90 + 120 + 145 = 799.

Clearly, sales are maximum in case of 32AH batteries.

2. (*d*) : Required difference = [(84 − 45) × 1000] = 39000.

3. (*d*) : The percentages of sales of 4AH batteries to the total sales in different years are :

$$\textbf{For 1992} = \left(\frac{75}{543} \times 100\right)\% = 13.81\%; \quad \textbf{For 1993} = \left(\frac{90}{528} \times 100\right)\% = 17.05\%;$$

$$\textbf{For 1994} = \left(\frac{96}{525} \times 100\right)\% = 18.29\%; \quad \textbf{For 1995} = \left(\frac{105}{510} \times 100\right)\% = 20.59\%;$$

$$\textbf{For 1996} = \left(\frac{96}{465} \times 100\right)\% = 19.35\%; \quad \textbf{For 1997} = \left(\frac{105}{495} \times 100\right)\% = 21.21\%;$$

$$\textbf{For 1998} = \left(\frac{115}{605} \times 100\right)\% = 19.01\%.$$

Clearly, the percentage is maximum in 1997.

4. (*b*) : From the table it is clear that the sales of 7AH batteries have been decreasing continuously from 1992 to 1997.

5. (*d*) : Required Percentage $= \left[\frac{(145 - 108)}{108} \times 100\right]\% = 34.26\% \approx 34\%.$

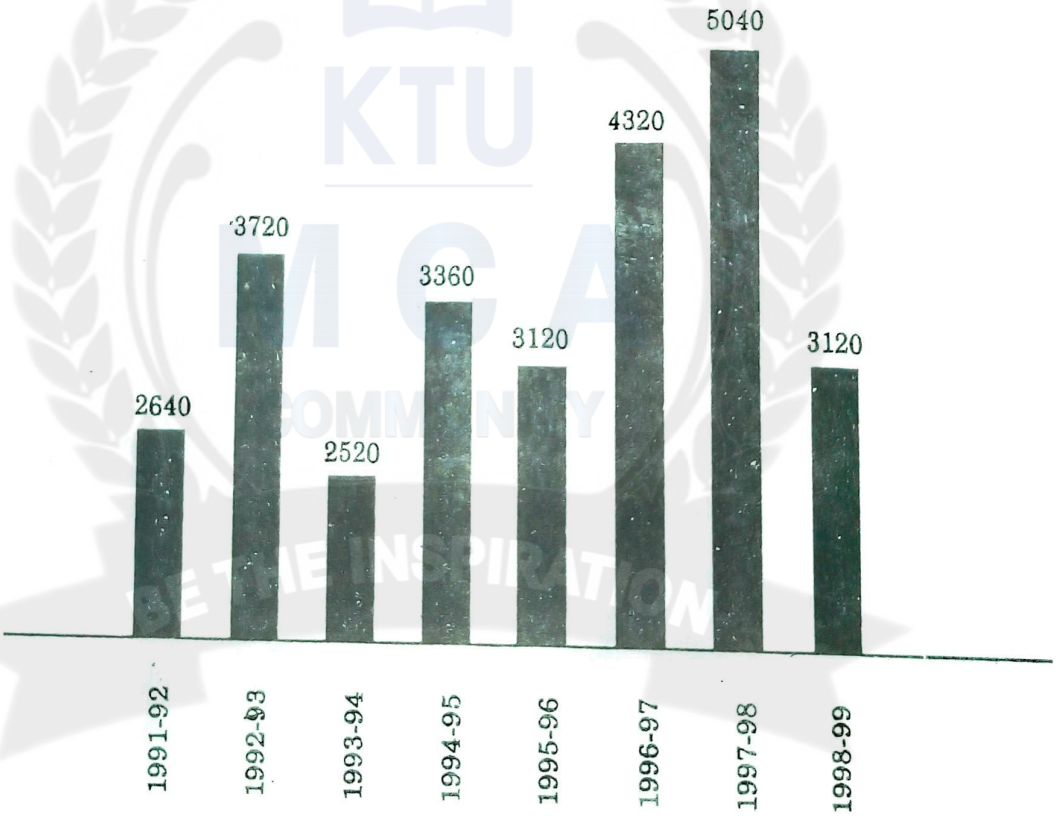
37. BAR GRAPHS

This section comprises of questions in which the data collected in a particular discipline are represented in the form of vertical or horizontal bars drawn by selecting a particular scale. One of the parameters is plotted on the horizontal axis and the other on the vertical axis. The candidate is required to understand the given information and thereafter answer the given questions on the basis of data analysis.

Ex. 1. ***The bar graph given below shows the foreign exchange reserves of a country (in million US $) from 1991-92 to 1998-99. Answer the questions based on this graph.*** (Bank P.O. 2001)

FOREIGN EXCHANGE RESERVES OF A COUNTRY

(in million US $)



1. The foreign exchange reserves in 1997-98 was how many times that in 1994-95 ?

   (*a*) 0.7 (*b*) 1.2 (*c*) 1.4 (*d*) 1.5 (*e*) 1.8

2. What was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94 ?

   (*a*) 100 (*b*) 150 (*c*) 200 (*d*) 620 (*e*) 2520

3 For which year, the percent increase of foreign exchange reserves over the previous year is the highest ?

   (*a*) 1992-93 (*b*) 1993-94 (*c*) 1994-95 (*d*) 1996-97 (*e*) 1997-98

4. The foreign exchange reserves in 1996-97 were approximately what percent of the average foreign exchange reserves over the period under review ?

(a) 95% (b) 110% (c) 115% (d) 125% (e) 140%

5. The ratio of the number of years, in which the foreign exchange reserves are above the average reserves, to those in which the reserves are below the average reserves, is :

(a) 2 : 6 (b) 3 : 4 (c) 3 : 5 (d) 4 : 4 (e) 5 : 3

Sol. 1. (d) : Required ratio $= \frac{5040}{3360} = 1.5.$

2. (a) : Foreign exchange reserves in 1997-98 = 5040 million US \$

Foreign exchange reserves in 1993-94 = 2520 million US \$.

$\therefore$ Increase = (5040 – 2520) = 2520 million US \$.

$\therefore$ Percentage increase $= \left(\frac{2520}{2520} \times 100\right)\% = 100\%.$

3. (a) : There is an increase in foreign exchange reserves during the years 1992-93, 1994-95, 1996-97 and 1997-98 as compared to previous year (as shown by bar-graph).

The percentage increase in reserves during these years compared to previous year are :

(*i*) For **1992-93** $= \left[\frac{(3720 - 2640)}{2640} \times 100\right]\% = 40.91\%$

(*ii*) For **1994-95** $= \left[\frac{(3360 - 2520)}{2520} \times 100\right]\% = 33.33\%$

(*iii*) For **1996-97** $= \left[\frac{(4320 - 3120)}{3120} \times 100\right]\% = 38.46\%$

(*iv*) For **1997-98** $= \left[\frac{(5040 - 4320)}{4320} \times 100\right]\% = 16.67\%$

Clearly, the percentage increase over previous year is highest for 1992-93.

4. (d) : Average foreign exchange reserves over the given period

$= \left[\frac{1}{8} \times (2640 + 3720 + 2520 + 3360 + 3120 + 4320 + 5040 + 3120)\right]$ million US \$

= 3480 million US \$.

Foreign exchange reserves in 1996-97 = 4320 million US \$.

$\therefore$ Required Percentage $= \left(\frac{4320}{3480} \times 100\right)\% = 124.14\% \approx 125\%.$

5. (c) : Average foreign exchange reserves over the given period = 3480 million US \$.

The country had reserves above 3480 million US \$ during the years 1992-93, 1996-97 and 1997-98 *i.e.*, for 3 years and below 3480 million US \$ during the years 1991-92, 1993-94, 1994-95, 1995-96 and 1998-99 *i.e.*, for 5 years.

Hence, required ratio = 3 : 5.

Ex. 2. The h

38. PIE-CHARTS

IMPORTANT FACTS AND FORMULAE

The **pie-chart** or a **pie-graph** is a method of representing a given numerical data in the form of sectors of a circle.

The sectors of the circle are constructed in such a way that the area of each sector is proportional to the corresponding value of the component of the data.

From geometry, we know that the area of the sector of a circle is proportional to the central angle.

So, the central angle of each sector must be proportional to the corresponding value of the component.

Since the sum of all the central angles is 360°, we have

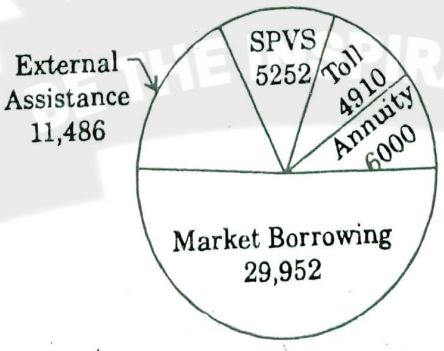
$$\text{Central angle of the component} = \left(\frac{\text{Value of the component}}{\text{Total value}} \times 360\right)^\circ$$

SOLVED EXAMPLES

The procedure of solving problems based on pie-charts will be clear from the following solved examples.

Example 1. ***The following pie-chart shows the sources of funds to be collected by the National Highways Authority of India (NHAI) for its Phase II projects. Study the pie-chart and answer the questions that follow.***

SOURCES OF FUNDS TO BE ARRANGED BY NHAI FOR PHASE II PROJECTS (IN CRORES RS.)



Total funds to be arranged for Projects (Phase II) = Rs. 57,600 crores.

1. Near about 20% of the funds are to be arranged through :
   (a) SPVS (b) External Assistance
   (c) Annuity (d) Market Borrowing
2. The central angle corresponding to Market Borrowing is :
   (a) 52° (b) 137.8% (c) 187.2° (d) 192.4°
3. The approximate ratio of the funds to be arranged through Toll and that through Market Borrowing is :
   (a) 2 : 9 (b) 1 : 6 (c) 3 : 11 (d) 2 : 5

4. If NHAI could receive a total of Rs. 9695 crores as External Assistance, by what percent (approximately) should it increase the Market Borrowings to arrange for the shortage of funds ?

(a) 4.5% (b) 7.5% (c) 6% (d) 8%

5. If the toll is to be collected through an outsourced agency by allowing a maximum 10% commission, how much amount should be permitted to be collected by the outsourced agency, so that the project is supported with Rs. 4910 crores ?

(a) Rs. 6213 crores (b) Rs. 5827 crores (c) Rs. 5401 crores (d) Rs. 5216 crores

SOLUTION

1. **(b)** : 20% of the total funds to be arranged = Rs. (20% of 57600) crores
= Rs. 11520 crores ≈ Rs. 11486 crores.

Rs. 11486 crores is the amount of funds to be arranged through External Assistance.

2. **(c)** : Central angle corresponding to Market Borrowing $= \left(\frac{29952}{57600} \times 360^\circ\right) = 187.2^\circ$

3. **(b)** : Required ratio $= \frac{4910}{29952} = \frac{1}{6.1} \approx \frac{1}{6}$.

4 **(c)** : Shortage of funds arranged through External Assistance
= Rs. (11486 − 9695) crores = Rs. 1791 crores.

∴ Increase required in Market Borrowings = Rs. 1791 crores.

Percentage increase required $= \left(\frac{1791}{29952} \times 100\right)\% = 5.98\% \approx 6\%$.

5. **(c)** : Amount permitted = (Funds required from Toll for projects of Phase II)
+ (10% of these funds)
= Rs. 4910 crores + Rs. (10% of 4910) crores
= Rs. (4910 + 491) crores = Rs. 5401 crores

Example 2. *The pie chart given below shows the distribution of the total*

39. LINE-GRAPHS

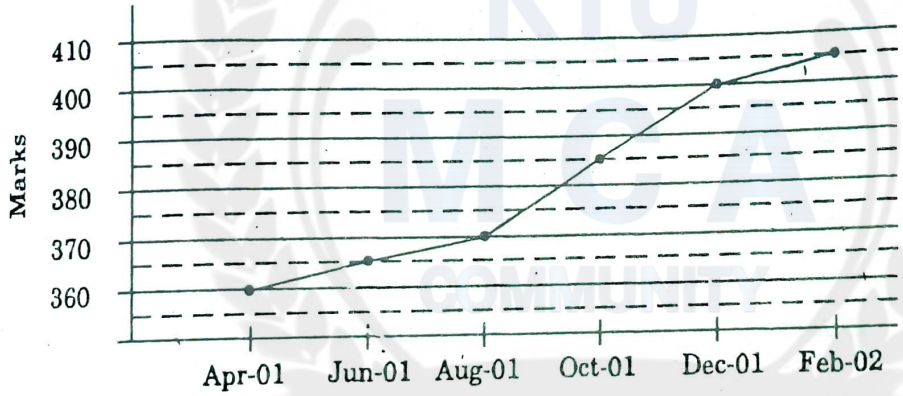
This section comprises of questions in which the data collected in a particular discipline are represented by specific points joined together by straight lines. The points are plotted on a two-dimensional plane taking one parameter on the horizontal axis and the other on the vertical axis. The candidate is required to analyse the given information and thereafter answer the given questions on the basis of the analysis of data.

SOLVED EXAMPLES

Ex. 1. ***In a school the periodical examinations are held every second month. In a session during Apr. 2001 – Mar. 2002, a student of Class IX appeared for each of the periodical exams. The aggregate marks obtained by him in each periodical exam are represented in the line-graph given below. Study the graph and answer the questions based on it.*** (S.B.I.P.O. 2003)

MARKS OBTAINED BY A STUDENT IN SIX PERIODICAL EXAMS HELD IN EVERY TWO MONTHS DURING THE YEAR IN THE SESSION 2001-02

Maximum Total Marks in each Periodical Exam = 500



1. The total number of marks obtained in Feb. 02 is what percent of the total marks obtained in Apr. 01 ?
   (a) 110% (b) 112.5% (c) 115% (d) 116.5% (e) 117.5%
2. What are the average marks obtained by the student in all the periodical exams during the session ?
   (a) 373 (b) 379 (c) 381 (d) 385 (e) 389
3. What is the percentage of marks obtained by the student in the periodical exams of Aug. 01 and Oct. 01 taken together ?
   (a) 73.25% (b) 75.5% (c) 77% (d) 78.75% (e) 79.5%
4. In which periodical exams there is a fall in percentage of marks as compared to the previous periodical exams ?
   (a) None (b) Jun. 01 (c) Oct. 01 (e) Feb. 02 (e) None of these
5. In which periodical exams did the student obtain the highest percentage increase in marks over the previous periodical exams ?
   (a) Jun. 01 (b) Aug. 01 (c) Oct. 01 (d) Dec. 01 (e) Feb. 02

Sol. Here it is clear from the graph that the student obtained 360, 365, 370, 385, 400 and 405 marks in periodical exams held in Apr. 01, Jun. 01, Aug. 01, Oct. 01, Dec. 01 and Feb. 02 respectively.

1. (*b*) : Required percentage $= \left(\frac{405}{360} \times 100\right)\% = 112.5\%$.

2. (*c*) : Average marks obtained in all the periodical exams

$$= \frac{1}{6} \times [360 + 365 + 370 + 385 + 400 + 405] = 380.83 \approx 381.$$

3. (*b*) : Required percentage $= \left[\frac{(370 + 385)}{(500 + 500)} \times 100\right]\% = \left(\frac{755}{1000} \times 100\right)\% = 75.5\%$.

4. (*a*) : As is clear from the graph, the total marks obtained in periodical exams, go on increasing. Since, the maximum marks for all the periodical exams are same, it implies that the percentage of marks also goes on increasing. Thus, in none of the periodical exams, there is a fall in percentage of marks compared to the previous exam.

5. (*c*) : Percentage increase in marks in various periodical exams compared to the previous exams are :

For **Jun. 01** $= \left[\frac{(365 - 360)}{360} \times 100\right]\% = 1.39\%$

For **Aug. 01** $= \left[\frac{(370 - 365)}{365} \times 100\right]\% = 1.37\%$

For **Oct. 01** $= \left[\frac{(385 - 370)}{370} \times 100\right]\% = 4.05\%$

For **Dec. 01** $= \left[\frac{(400 - 385)}{385} \times 100\right]\% = 3.90\%$

For **Feb. 02** $= \left[\frac{(405 - 400)}{400} \times 100\right]\% = 1.25\%$.

Clearly, highest percentage increase in marks is in Oct. 01.

Ex. 2. *The following line-graph gives the ratio of the amounts of imports by a*