23. LOGARITHMS

IMPORTANT FACTS AND FORMULAE

I. **Logarithm** : If a is a positive real number, other than 1 and $a^m = x$, then we write : $m = \log_a x$ and we say that the value of log x to the base a is m.

Example :

(i) $10^3 = 1000 \Rightarrow \log_{10} 1000 = 3$ (ii) $3^4 = 81 \Rightarrow \log_3 81 = 4$

(iii) $2^{-3} = \frac{1}{8} \Rightarrow \log_2 \frac{1}{8} = -3$ (iv) $(.1)^2 = .01 \Rightarrow \log_{(.1)} .01 = 2$.

II. **Properties of Logarithms :**

1. $\log_a (xy) = \log_a x + \log_a y$
2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_x x = 1$
4. $\log_a 1 = 0$
5. $\log_a (x^p) = p (\log_a x)$
6. $\log_a x = \frac{1}{\log_x a}$
7. $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a}$.

Remember : When base is not mentioned, it is taken as 10.

III. **Common Logarithms** : Logarithms to the base 10 are known as common logarithms.

IV. The logarithm of a number contains two parts, namely *characteristic* and *mantissa*.

Characteristic : The integral part of the logarithm of a number is called its ***characteristic***.

Case I : When the number is greater than 1.

In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

Case II : When the number is less than 1.

In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.

Instead of − 1, − 2, etc. we write, $\bar{1}$ (one bar), $\bar{2}$ (two bar), etc.

Example :

Number	Characteristic	Number	Characteristic
348.25	2	0.6173	$\bar{1}$
46.583	1	0.03125	$\bar{2}$
9.2193	0	0.00125	$\bar{3}$

Mantissa : The decimal part of the logarithm of a number is known is its ***mantissa***.

For mantissa, we look through log table.

SOLVED EXAMPLES

Ex. 1. *Evaluate : (i)* $\log_3 27$ *(ii)* $\log_7 \left(\frac{1}{343}\right)$ *(iii)* $\log_{100} (0.01)$

Sol. (i) Let $\log_3 27 = n$.

Then, $3^n = 27 = 3^3$ or $n = 3$. $\therefore \log_3 27 = 3$.

(ii) Let $\log_7 \left(\frac{1}{343}\right) = n$.

Then, $7^n = \frac{1}{343} = \frac{1}{7^3} = 7^{-3}$ or $n = -3$. $\therefore \log_7 \left(\frac{1}{343}\right) = -3$.

(iii) Let $\log_{100} (0.01) = n$.

Then, $(100)^n = 0.01 = \frac{1}{100} = (100)^{-1}$ or $n = -1$ $\therefore \log_{100} (0.01) = -1$.

Ex. 2. *Evaluate : (i)* $\log_7 1 = 0$ *(ii)* $\log_{34} 34$ *(iii)* $36^{\log_6 4}$

Sol. (i) We know that $\log_a 1 = 0$, so $\log_7 1 = 0$.

(ii) We know that $\log_a a = 1$, so $\log_{34} 34 = 0$.

(iii) We know that $a^{\log_a x} = x$.

Now, $36^{\log_6 4} = (6^2)^{\log_6 4} = 6^{2(\log_6 4)} = 6^{\log_6 (4^2)} = 6^{\log_6 16} = 16$.

Ex. 3. *If* $\log_{\sqrt{8}} x = 3\frac{1}{3}$*, find the value of x.*

Sol. $\log_{\sqrt{8}} x = \frac{10}{3} \Leftrightarrow x = (\sqrt{8})^{10/3} = (2^{3/2})^{10/3} = 2^{\left(\frac{3}{2} \times \frac{10}{3}\right)} = 2^5 = 32$.

Ex. 4. *Evaluate : (i)* $\log_5 3 \times \log_{27} 25$ *(ii)* $\log_9 27 - \log_{27} 9$

Sol. (i) $\log_5 3 \times \log_{27} 25 = \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 27} = \frac{\log 3}{\log 5} \times \frac{\log (5^2)}{\log (3^3)} = \frac{\log 3}{\log 5} \times \frac{2 \log 5}{3 \log 3} = \frac{2}{3}$.

(ii) Let $\log_9 27 = n$.

Then, $9^n = 27 \Leftrightarrow 3^{2n} = 3^3 \Leftrightarrow 2n = 3 \Leftrightarrow n = \frac{3}{2}$.

Again, let $\log_{27} 9 = m$.

Then, $27^m = 9 \Leftrightarrow 3^{3m} = 3^2 \Leftrightarrow 3m = 2 \Leftrightarrow m = \frac{2}{3}$.

$\therefore \log_9 27 - \log_{27} 9 = (n - m) = \left(\frac{3}{2} - \frac{2}{3}\right) = \frac{5}{6}$.

Ex. 5. *Simplify :* $\left(\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243}\right)$ (S.S.C. 2000)

Sol. $\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243} = \log \frac{75}{16} - \log \left(\frac{5}{9}\right)^2 + \log \frac{32}{243} = \log \frac{75}{16} - \log \frac{25}{81} + \log \frac{32}{243}$

$$= \log \left(\frac{75}{16} \times \frac{32}{243} \times \frac{81}{25}\right) = \log 2.$$

24. AREA

FUNDAMENTAL CONCEPTS

I. **Results on Triangles :**

1. Sum of the angles of a triangle is 180°.
2. The sum of any two sides of a triangle is greater than the third side.
3. **Pythagoras Theorem :** In a right-angled triangle,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2.$$

4. The line joining the mid-point of a side of a triangle to the opposite vertex is called the ***median***.
5. The point where the three medians of a triangle meet, is called ***centroid***. The centroid divides each of the medians in the ratio 2 : 1.
6. In an isosceles triangle, the altitude from the vertex bisects the base.
7. The median of a triangle divides it into two triangles of the same area.
8. The area of the triangle formed by joining the mid-points of the sides of a given triangle is one-fourth of the area of the given triangle.

II. **Results on Quadrilaterals :**

1. The diagonals of a parallelogram bisect each other.
2. Each diagonal of a parallelogram divides it into two triangles of the same area.
3. The diagonals of a rectangle are equal and bisect each other.
4. The diagonals of a square are equal and bisect each other at right angles.
5. The diagonals of a rhombus are unequal and bisect each other at right angles.
6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
7. Of all the parallelogram of given sides, the parallelogram which is a rectangle has the greatest area.

IMPORTANT FORMULAE

I. 1. Area of a rectangle = (Length × Breadth).

$$\therefore \quad \text{Length} = \left(\frac{\text{Area}}{\text{Breadth}}\right) \text{ and Breadth} = \left(\frac{\text{Area}}{\text{Length}}\right).$$

2. Perimeter of a rectangle = 2 (Length + Breadth).

II. Area of a square $= (\text{side})^2 = \frac{1}{2}(\text{diagonal})^2$.

III. Area of 4 walls of a room = 2 (Length + Breadth) × Height.

IV. 1. Area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$.

2. Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c are the sides of the triangle and $s = \frac{1}{2}(a+b+c)$.

3. Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

4. Radius of incircle of an equilateral triangle of side $a = \frac{a}{2\sqrt{3}}$.

5. Radius of circumcircle of an equilateral triangle of side $a = \frac{a}{\sqrt{3}}$.

6. Radius of incircle of a triangle of area Δ and semi-perimeter $s = \frac{\Delta}{s}$.

V. 1. Area of a parallelogram = (Base × Height).

2. Area of a rhombus $= \frac{1}{2} \times$ (Product of diagonals).

3. Area of a trapezium $= \frac{1}{2} \times$ (sum of parallel sides) × distance between them.

VI. 1. Area of a circle $= \pi R^2$, where R is the radius.

2. Circumference of a circle $= 2\pi R$.

3. Length of an arc $= \frac{2\pi R\theta}{360}$, where θ is the central angle.

4. Area of a sector $= \frac{1}{2}(\text{arc} \times R) = \frac{\pi R^2 \theta}{360}$.

VII. 1. Area of a semi-circle $= \frac{\pi R^2}{2}$

2. Circumference of a semi-circle $= \pi R$.

SOLVED EXAMPLES

Ex. 1. ***One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.***

Sol. Other side $= \sqrt{(17)^2 - (15)^2} = \sqrt{289 - 225} = \sqrt{64} = 8$ m.

$\therefore$ Area $= (15 \times 8)\ m^2 = 120\ m^2$.

Ex. 2. ***A lawn is in the form of a rectangle having its sides in the ratio 2 : 3. The area of the lawn is $\frac{1}{6}$ hectares. Find the length and breadth of the lawn.***

Sol. Let length $= 2x$ metres and breadth $= 3x$ metres.

Now, area $= \left(\frac{1}{6} \times 1000\right) m^2 = \left(\frac{5000}{3}\right) m^2$.

So, $2x \times 3x = \frac{5000}{3} \Leftrightarrow x^2 = \frac{2500}{9} \Leftrightarrow x = \left(\frac{50}{3}\right)$.

$\therefore$ Length $= 2x = \frac{100}{3}$ m $= 33\frac{1}{3}$ m and Breadth $= 3x = \left(3 \times \frac{50}{3}\right)$ m $= 50$ m.

Ex. 3. ***Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per square metre.***

Sol. Area of the carpet = Area of the room $= (13 \times 9)\ m^2 = 117\ m^2$.

Length of the carpet $= \left(\frac{\text{Area}}{\text{Width}}\right) = \left(117 \times \frac{4}{3}\right)$ m $= 156$ m.

$\therefore$ Cost of carpeting = Rs. (156×12.40) = Rs. 1934.40.

25. VOLUME AND SURFACE AREA

IMPORTANT FORMULAE

I. CUBOID

Let length = l, breadth = b and height = h units. Then,

1. ***Volume*** = $(l \times b \times h)$ cubic units.
2. ***Surface area*** = $2(lb + bh + lh)$ sq. units.
3. ***Diagonal*** = $\sqrt{l^2 + b^2 + h^2}$ units.

II. CUBE

Let each edge of a cube be of length a. Then,

1. ***Volume*** = a^3 cubic units.
2. ***Surface area*** = $6a^2$ sq. units.
3. ***Diagonal*** = $\sqrt{3}\,a$ units.

III. CYLINDER

Let radius of base = r and Height (or length) = h. Then,

1. ***Volume*** = $(\pi r^2 h)$ cubic units.
2. ***Curved surface area*** = $(2\pi rh)$ sq. units.
3. ***Total surface area*** = $(2\pi rh + 2\pi r^2)$ sq. units

   $= 2\pi r(h + r)$ sq. units.

IV. CONE

Let radius of base = r and Height = h. Then,

1. ***Slant height,*** $l = \sqrt{h^2 + r^2}$ units.
2. ***Volume*** = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.
3. ***Curved surface area*** = (πrl) sq. units.
4. ***Total surface area*** = $(\pi rl + \pi r^2)$ sq. units.

V. SPHERE

Let the radius of the sphere be r. Then,

1. ***Volume*** = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.
2. ***Surface area*** = $(4\pi r^2)$ sq. units.

VI. HEMISPHERE

Let the radius of a hemisphere be r. Then,

1. ***Volume*** = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.
2. ***Curved surface area*** = $(2\pi r^2)$ sq. units.
3. ***Total surface area*** = $(3\pi r^2)$ sq. units.

Remember : 1 litre = 1000 cm^3.

SOLVED EXAMPLES

Ex. 1. *Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.*

Sol. Volume = $(16 \times 14 \times 7)\ m^3 = 1568\ m^3$

Surface area = $[2(16 \times 14 + 14 \times 7 + 16 \times 7)]\ cm^2 = (2 \times 434)\ cm^2 = 868\ cm^2$

Ex. 2. *Find the length of the longest pole that can be placed in a room 12 m long, 8 m broad and 9 m high.*

Sol. Length of longest pole = Length of the diagonal of the room

$= \sqrt{(12)^2 + 8^2 + 9^2} = \sqrt{289} = 17$ m.

Ex. 3. *The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.*

Sol. Let the breadth of the wall be x metres.

Then, Height = $5x$ metres and, Length = $40x$ metres.

$\therefore \quad x \times 5x \times 40x = 12.8 \Leftrightarrow x^3 = \frac{12.8}{200} = \frac{128}{2000} = \frac{64}{1000}$.

So, $x = \frac{4}{10}$ m $= \left(\frac{4}{10} \times 100\right)$ cm = 40 cm.

Ex. 4. *Find the number of bricks, each measuring 24 cm × 12 cm × 8 cm, required to construct a wall 24 m long, 8m high and 60 cm thick, if 10% of the wall is filled with mortar?*

Sol. Volume of the wall = $(2400 \times 800 \times 60)$ cu. cm.

Volume of bricks = 90% of the volume of the wall

$= \left(\frac{90}{100} \times 2400 \times 800 \times 60\right)$ cu. cm.

Volume of 1 brick = $(24 \times 12 \times 8)$ cu. cm.

$\therefore$ Number of bricks = $\left(\frac{90}{100} \times \frac{2400 \times 800 \times 60}{24 \times 12 \times 8}\right) = 45000$.

Ex. 5. *Water flows into a tank 200 m × 150 m through a rectangular pipe 1.5 m × 1.25 m @ 20 kmph. In what time (in minutes) will the water rise by 2 metres?*

Sol. Volume required in the tank = $(200 \times 150 \times 2)\ m^3 = 60000\ m^3$.

Length of water column flown in 1 min. = $\left(\frac{20 \times 1000}{60}\right)$ m $= \frac{1000}{3}$ m.

Volume flown per minute = $\left(1.5 \times 1.25 \times \frac{1000}{3}\right) m^3 = 625\ m^3$.

$\therefore$ Required time = $\left(\frac{60000}{625}\right)$ min. = 96 min.

Ex. 6. *The dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 3 cm. If 1 cubic cm of metal used in the box weighs 0.5 gms, find the weight of the box.*

Sol. Volume of the metal used in the box = External Volume − Internal Volume

$= [(50 \times 40 \times 23) - (44 \times 34 \times 20)]\ cm^3$

$= 16080\ cm^3$.

$\therefore$ Weight of the metal = $\left(\frac{16080 \times 0.5}{1000}\right)$ kg = 8.04 kg.

26. RACES AND GAMES OF SKILL

IMPORTANT FACTS

Races : *A contest of speed in running, riding, driving, sailing or rowing is called a race.*

Race Course : *The ground or path on which contests are made is called a race course.*

Starting Point : *The point from which a race begins is known as a starting point.*

Winning Point or Goal : *The point set to bound a race is called a winning point or a goal.*

Winner : *The person who first reaches the winning point is called a winner.*

Dead Heat Race : *If all the persons contesting a race reach the goal exactly at the same time, then the race is said to be a dead heat race.*

Start : Suppose A and B are two contestants in a race. If before the start of the race, A is at the starting point and B is ahead of A by 12 metres, then we say that '*A gives B, a start of 12 metres*'.

To cover a race of 100 metres in this case, A will have to cover 100 metres while B will have to cover only (100 − 12) = 88 metres.

In a 100 m race, '*A can give B 12 m*' or '*A can give B a start of 12 m*' or '*A beats B by 12 m*' means that while A runs 100 m, B runs (100 − 12) = 88 m.

Games : '*A game of 100, means that the person among the contestants who scores 100 points first is the winner*'.

If A scores 100 points while B scores only 80 points, then we say that '*A can give B 20 points*'.

SOLVED EXAMPLES

Ex. 1. *In a km race, A beats B by 28 metres or 7 seconds. Find A's time over the course.*

Sol. Clearly, B covers 28 m in 7 seconds.

∴ B's time over the course $= \left(\frac{7}{28} \times 1000\right)$ sec = 250 seconds.

∴ A's time over the course = (250 − 7) sec = 243 sec = 4 min. 3 sec.

Ex. 2. *A runs $1\frac{3}{4}$ times as fast as B. If A gives B a start of 84 m, how far must the winning post be so that A and B might reach it at the same time ?*

Sol. Ratio of the rates of A and B $= \frac{7}{4} : 1 = 7 : 4$.

So, in a race of 7 m, A gains 3 m over B.

∴ 3 m are gained by A in a race of 7 m

∴ 84 m are gained by A in a race of $\left(\frac{7}{3} \times 84\right)$ m = 196 m

∴ Winning post must be 196 m away from the starting point.

27. CALENDAR

IMPORTANT FACTS AND FORMULAE

We are supposed to find the day of the week on a given date.

For this, we use the concept of *odd days*.

I. Odd Days : In a given period, the number of days more than the complete weeks are called *odd days*.

II. Leap Year :

(*i*) Every year divisible by 4 is a leap year, if it is not a century.

(*ii*) Every 4th century is a leap year and no other century is a leap year.

Note : A leap year has **366** days.

Examples :

(*i*) Each of the years 1948, 2004, 1676 etc. is a leap year.

(*ii*) Each of the years 400, 800, 1200, 1600, 2000 etc. is a leap year.

(*iii*) None of the years 2001, 2002, 2003, 2005, 1800, 2100 is a leap year.

III. Ordinary Year :

The year which is not a leap year is called an ordinary year. An ordinary year has 365 days.

IV. Counting of Odd Days :

(*i*) 1 ordinary year = 365 days = (52 weeks + 1 day).

∴ *1 ordinary year has 1 odd day.*

(*ii*) 1 leap year = 366 days = (52 weeks + 2 days).

∴ *1 leap year has 2 odd days.*

(*iii*) 100 years = 76 ordinary years + 24 leap years
= (76 × 1 + 24 × 2) odd days = 124 odd days
= (17 weeks + 5 days) ≡ 5 odd days.

∴ Number of odd days in 100 years = 5
Number of odd days in 200 years = (5 × 2) ≡ 3 odd days
Number of odd days in 300 years = (5 × 3) ≡ 1 odd day.
Number of odd days in 400 years = (5 × 4 + 1) ≡ 0 odd day.
Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years etc. has 0 odd day.

V. Day of the Week Related to Odd Days:

No. of days	0	1	2	3	4	5	6
Day	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.

SOLVED EXAMPLES

Ex. 1. ***What was the day of the week on 16th July, 1776 ?***

Sol. 16th July, 1776 = (1775 years + Period from 1.1.1776 to 16.7.1776)

Counting of odd days :

Number of odd days in 1600 years = 0
Number of odd days in 100 years = 5
75 years = 18 leap years + 57 ordinary years
= (18 × 2 + 57 × 1) odd days = 93 odd days
= (13 weeks + 2 days) ≡ 2 odd days

∴ 1775 years have = (0 + 5 + 2) odd days = 7 odd days ≡ 0 odd day.

Jan.		Feb.		March		April		May		June		July	
(31	+	29	+	31	+	30	+	31	+	30	+	16)	= 198 days

198 days = (28 weeks + 2 days) ≡ 2 odd days.

∴ Total number of odd days = (0 + 2) = 2.

Hence, the required day is Tuesday.

28. CLOCKS

IMPORTANT FACTS

The face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.

A clock has two hands, the smaller one is called the *hour hand* or *short hand* while the larger one is called the *minute hand* or *long hand*.

(*i*) In 60 minutes, the minute hand gains 55 minutes on the hour hand.

(*ii*) In every hour, both the hands coincide once.

(*iii*) The hands are in the same straight line when they are coincident or opposite to each other.

(*iv*) When the two hands are at right angles, they are 15 minute spaces apart.

(*v*) When the hands are in opposite directions, they are 30 minute spaces apart.

(*vi*) Angle traced by hour hand in 12 hrs = 360°.

(*vii*) Angle traced by minute hand in 60 min. = 360°.

Too Fast and Too Slow : *If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast.*

On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow.

SOLVED EXAMPLES

Ex. 1. ***Find the angle between the hour hand and the minute hand of a clock when the time is 3.25.***

Sol. Angle traced by the hour hand in 12 hours = 360°.

Angle traced by it in 3 hrs 25 min. *i.e.* $\frac{41}{12}$ hrs = $\left(\frac{360}{12} \times \frac{41}{12}\right)^\circ = 102\frac{1}{2}^\circ$

Angle traced by minute hand in 60 min. = 360°.

Angle traced by it in 25 min. = $\left(\frac{360}{60} \times 25\right)^\circ = 150°$

∴ Required angle = $\left(150° - 102\frac{1}{2}^\circ\right) = 47\frac{1}{2}^\circ$

Ex. 2. ***At what time between 2 and 3 o'clock will the hands of a clock be together?***

Sol. At 2 o'clock, the hour hand is at 2 and the minute hand is at 12, *i.e.* they are 10 min. spaces apart.

To be together, the minute hand must gain 10 minutes over the hour hand.

Now, 55 minutes are gained by it in 60 min.

∴ 10 minutes will be gained in $\left(\frac{60}{55} \times 10\right)$ min. = $10\frac{10}{11}$ min.

∴ The hands will coincide at $10\frac{10}{11}$ min. past 2.

29. STOCK AND SHARES

To start a big business or an industry, a large amount of money is needed. It is beyond the capacity of one or two persons to arrange such a huge amount. However, some persons associate together to form a company. They, then, draft a proposal, issue a prospectus (in the name of the company), explaining the plan of the project and invite the public to invest money in this project. They, thus, pool up the funds from the public, by assigning them ***shares*** of the company.

IMPORTANT FACTS AND FORMULAE

1. **Stock-capital :** *The total amount of money needed to run the company is called the* ***stock-capital.***
2. **Shares or Stock :** *The whole capital is divided into small units, called* ***shares*** *or* ***stock.***

   For each investment, the company issues a *share-certificate*, showing the value of each share and the number of shares held by a person.

   The person who subscribes in shares or stock is called a ***share holder*** *or* ***stock holder.***
3. **Dividend :** *The annual profit distributed among share holders is called* ***dividend.***

   Dividend is paid annually as per share or as a percentage.
4. **Face Value :** *The value of a share or stock printed on the share-certificate is called its* ***Face Value*** *or* ***Nominal Value*** *or* ***Par Value.***
5. **Market Value :** The stocks of different companies are sold and bought in the open market through brokers at stock-exchanges. A share (or stock) is said to be :

   (*i*) ***At premium*** or ***Above par,*** if its market value is more than its face value.

   (*ii*) ***At par,*** if its market value is the same as its face value.

   (*iii*) ***At discount*** or ***Below par,*** if its market value is less than its face value.

   Thus, if a Rs. 100 stock is quoted at a premium of 16, then market value of the stock = Rs. (100 + 16) = Rs. 116.

   Likewise, if a Rs. 100 stock is quoted at a discount of 7, then market value of the stock = Rs. (100 − 7) = Rs. 93.
6. **Brokerage :** *The broker's charge is called* ***brokerage.***

   (*i*) When stock is purchased, brokerage is added to the cost price.

   (*ii*) When stock is sold, brokerage is subtracted from the selling price.

Remember :

(*i*) The face value of a share always remains the same.

(*ii*) The market value of a share changes from time to time.

(*iii*) Dividend is always paid on the face value of a share.

(*iv*) Number of shares held by a person

$$= \frac{\text{Total Investment}}{\text{Investment in 1 share}} = \frac{\text{Total Income}}{\text{Income from 1 share}} = \frac{\text{Total Face Value}}{\text{Face value of 1 share}}.$$

Thus, by a Rs. 100, 9% stock at 120, we mean that :

(i) Face Value (N.V.) of stock = Rs. 100.

(ii) Market Value (M.V.) of stock = Rs. 120.

(iii) Annual dividend on 1 share = 9% of face value = 9% of Rs. 100 = Rs. 9.

(iv) An investment of Rs. 120 gives an annual income of Rs. 9.

(v) Rate of interest p.a. = Annual income from an investment of Rs. 100

$$= \left(\frac{9}{120} \times 100\right)\% = 7\frac{1}{2}\%.$$

SOLVED EXAMPLES

Ex. 1. ***Find the cost of :***

(i) Rs. 7200, 8% stock at 90;

(ii) Rs. 4500, 8.5% stock at 4 premium;

(iii) Rs. 6400, 10% stock at 15 discount.

Sol. (i) Cost of Rs. 100 stock = Rs. 90.

Cost of Rs. 7200 stock = Rs. $\left(\frac{90}{100} \times 7200\right)$ = Rs. 6480.

(ii) Cost of Rs. 100 stock = Rs. (100 + 4) = Rs. 104.

Cost of Rs. 4500 stock = Rs. $\left(\frac{104}{100} \times 4500\right)$ = Rs. 4680.

(iii) Cost of Rs. 100 stock = Rs. (100 − 15) = Rs. 85.

Cost of Rs. 6400 stock = Rs. $\left(\frac{85}{100} \times 6400\right)$ = Rs. 5440.

Ex. 2. ***Find the cash required to purchase Rs. 3200, $7\frac{1}{2}\%$ stock at 107 $\left(\text{brokerage } \frac{1}{2}\%\right)$.***

Sol. Cash required to purchase Rs. 100 stock = Rs. $\left(107 + \frac{1}{2}\right)$ = Rs. $\frac{215}{2}$.

Cash required to purchase Rs. 3200 stock = Rs. $\left(\frac{215}{2} \times \frac{1}{100} \times 3200\right)$ = Rs. 3440.

Ex. 3. ***Find the cash realised by selling Rs. 2440, 9.5% stock at 4 discount $\left(\text{brokerage } \frac{1}{4}\%\right)$.***

Sol. By selling Rs. 100 stock, cash realised = Rs. $\left[(100 - 4) - \frac{1}{4}\right]$ = Rs. $\frac{383}{4}$.

By selling Rs. 2400 stock, cash realised = Rs. $\left(\frac{383}{4} \times \frac{1}{100} \times 2400\right)$ = Rs. 2298.

30. PERMUTATIONS AND COMBINATIONS

IMPORTANT FACTS AND FORMULAE

Factorial Notation : Let n be a positive integer. Then, factorial n, denoted by $\underline{|n}$ or $n!$ is defined as

$$n! = n(n-1)(n-2) \ldots\ldots 3.2.1.$$

Examples : (*i*) $5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$; (*ii*) $4! = (4 \times 3 \times 2 \times 1) = 24$ etc.

We define, $0! = 1$.

Permutations : *The different arrangements of a given number of things by taking some or all at a time, are called permutations.*

Ex. 1. All permutations (or arrangements) made with the letters a, b, c by taking two at a time are (***ab, ba, ac, ca, bc, cb***).

Ex. 2. All permutations made with the letters a, b, c, taking all at a time are : (***abc, acb, bac, bca, cab, cba***).

Number of Permutations : Number of all permutations of n things, taken r at a time, is given by :

$$^nP_r = n(n-1)(n-2) \ldots\ldots (n-r+1) = \frac{n!}{(n-r)!}$$

Examples : (*i*) $^6P_2 = (6 \times 5) = 30$. (*ii*) $^7P_3 = (7 \times 6 \times 5) = 210$.

Cor. *Number of all permutations of n things, taken all at a time* $= n!$

An Important Result : *If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of rth kind, such that $(p_1 + p_2 + \ldots\ldots + p_r) = n$.*

Then, ***number of permutations of these n objects is :***

$$\frac{n!}{(p_1!)\cdot(p_2!)\ldots\ldots(p_r!)}.$$

Combinations : *Each of the different groups or selections which can be formed by taking some or all of a number of objects, is called a combination.*

Ex. 1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.

Note that AB and BA represent the same selection.

Ex. 2. All the combinations formed by a, b, c, taking two at a time are ***ab, bc, ca.***

Ex. 3. The only combination that can be formed of three letters a, b, c taken all at a time is ***abc.***

Ex. 4. Various groups of 2 out of four persons A, B, C, D are :

AB, AC, AD, BC, BD, CD.

Ex. 5. Note that *ab* and *ba* are two different permutations but they represent the same combination.

Number of Combinations : The number of all combinations of n things, taken r at a time is :

$$^{n}C_r = \frac{n!}{(r!)(n-r)!} = \frac{n(n-1)(n-2).....\text{to } r \text{ factors}}{r!}.$$

Note that : $^{n}C_n = 1$ *and* $^{n}C_0 = 1$.

An Important Result : $^{n}C_r = {}^{n}C_{(n-r)}$.

Example : (*i*) $^{11}C_4 = \frac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330.$

(*ii*) $^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_3 = \frac{16 \times 15 \times 14}{3!} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560.$

SOLVED EXAMPLES

Ex. 1. ***Evaluate :*** $\frac{30!}{28!}$

Sol. We have, $\frac{30!}{28!} = \frac{30 \times 29 \times (28!)}{28!} = (30 \times 29) = 870.$

Ex. 2. ***Find the value of*** (*i*) $^{60}P_3$ (*ii*)

Sol. (*i*) $^{60}P_3 = \frac{60!}{(60-3)!} = \frac{60!}{57!} = \frac{60 \times 59 \times 58 \times (57!)}{57!} = (60 \times 59 \times 58) = 205320.$

(*ii*) $^{4}P_4 = 4! = (4 \times 3 \times 2 \times 1) = 24.$

Ex. 3. ***Find the value of*** (*i*) $^{10}C_3$ (*ii*) $^{100}C_{98}$ (*iii*) $^{50}C_{50}$

Sol. (*i*) $^{10}C_3 = \frac{10 \times 9 \times 8}{3!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120.$

(*ii*) $^{100}C_{98} = {}^{100}C_{(100-98)} = {}^{100}C_2 = \left(\frac{100 \times 99}{2 \times 1}\right) = 4950.$

(*iii*) $^{50}C_{50} = 1.$ [$\because$ $^{n}C_n = 1$]

Ex. 4. ***How many words can be formed by using all letters of the word 'BIHAR' ?***

Sol. The word BIHAR contains 5 different letters.

$\therefore$ Required number of words = $^{5}P_5 = 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120.$

Ex. 5. ***How many words can be formed by using all the letters of the word 'DAUGHTER' so that the vowels always come together ?***

Sol. Given word contains 8 different letters. When the vowels AUE are always together, we may suppose them to form an entity, treated as one letter.

Then, the letters to be arranged are DGHTR (AUE).

These 6 letters can be arranged in $^{6}P_6 = 6! = 720$ ways.

The vowels in the group (AUE) may be arranged in $3! = 6$ ways.

$\therefore$ Required number of words = $(720 \times 6) = 4320.$

31. PROBABILITY

IMPORTANT FACTS AND FORMULAE

1. **Experiment** : An operation which can produce some well-defined outcomes is called an experiment.

2. **Random Experiment** : An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

   Examples of Performing a Random Experiment :

   (*i*) Rolling an unbiased dice.

   (*ii*) Tossing a fair coin.

   (*iii*) Drawing a card from a pack of well-shuffled cards.

   (*iv*) Picking up a ball of certain colour from a bag containing balls of different colours.

   Details :

   (*i*) When we throw a coin. Then either a Head (H) or a Tail (T) appears

   (*ii*) A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.

   (*iii*) A pack of cards has 52 cards.

   It has 13 cards of each suit, namely **Spades, Clubs, Hearts** and **Diamonds.**

   Cards of spades and clubs are **black cards.**

   Cards of hearts and diamonds are **red cards.**

   There are 4 honours of each suit.

   These are **Aces, Kings, Queens** and **Jacks.**

   These are called **face cards.**

3. **Sample Space** : When we perform an experiment, then the set S of all possible outcomes is called the **Sample Space.**

   Examples of Sample Spaces :

   (*i*) In tossing a coin, S = {H, T}.

   (*ii*) If two coins are tossed, then S = {HH, HT, TH, TT}.

   (*iii*) In rolling a dice, we have, S = {1, 2, 3, 4, 5, 6}.

4. **Event** : Any subset of a sample space is called an event.

5. **Probability of Occurrence of an Event :**

   Let S be the sample space and let E be an event.

   Then, $E \subseteq S$.

   $$\therefore \quad P(E) = \frac{n(E)}{n(S)}.$$

6. **Results on Probability :**

   (*i*) P (S) = 1 (*ii*) $0 \le P(E) \le 1$ (*iii*) $P(\phi) = 0$

   (*iv*) For any events A and B, we have :

   $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

   (*v*) If $\bar{A}$ denotes (not-A), then $P(\bar{A}) = 1 - P(A)$.

SOLVED EXAMPLES

Ex. 1. *In a throw of a coin, find the probability of getting a head.*

Sol. Here S = {H, T} and E = {H}.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{1}{2}$.

Ex. 2. *Two unbiased coins are tossed. What is the probability of getting at most one head ?*

Sol. Here S = {HH, HT, TH, TT}.

Let E = event of getting at most one head.

$\therefore$ E = {TT, HT, TH}.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$.

Ex. 3. *An unbiased die is tossed. Find the probability of getting a multiple of 3.*

Sol. Here S = {1, 2, 3, 4, 5, 6}.

Let E be the event of getting a multiple of 3.

Then, E = {3, 6}

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$.

Ex. 4. *In a simultaneous throw of a pair of dice, find the probability of getting a total more than 7.*

Sol. Here, n (S) = (6 × 6) = 36.

Let E = Event of getting a total more than 7

= {(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$.

Ex. 5. *A bag contains 6 white and 4 black balls. Two balls are drawn at random. Find the probability that they are of the same colour.*

Sol. Let S be the sample space. Then,

n (S) = Number of ways of drawing 2 balls out of $(6 + 4) = {}^{10}C_2 = \frac{(10 \times 9)}{(2 \times 1)} = 45$.

Let E = Event of getting both balls of the same colour. Then,

n (E) = Number of ways of drawing (2 balls out of 6) or (2 balls out of 4)

$= ({}^6C_2 + {}^4C_2) = \frac{(6 \times 5)}{(2 \times 1)} + \frac{(4 \times 3)}{(2 \times 1)} = (15 + 6) = 21$.

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{21}{45} = \frac{7}{15}$.

Ex. 6. *Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is divisible by 4 or 6 ?*

Sol. Clearly, n (S) = 6 × 6 = 36.

Let E be the event that the sum of the numbers on the two faces is divisible by 4 or 6. Then

E = {(1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (6, 2), (6, 6)}

$\therefore$ n (E) = 14.

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{14}{36} = \frac{7}{18}$.