17. TIME AND DISTANCE

IMPORTANT FACTS AND FORMULAE

1. Speed $= \left(\frac{\text{Distance}}{\text{Time}}\right)$, Time $= \left(\frac{\text{Distance}}{\text{Speed}}\right)$, Distance $=$ (Speed $\times$ Time)

2. x km/hr $= \left(x \times \frac{5}{18}\right)$ m/sec

3. x m/sec $= \left(x \times \frac{18}{5}\right)$ km/hr

4. If the ratio of the speeds of A and B is $a : b$, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or $b : a$.

5. Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr. Then, the average speed during the whole journey is $\left(\frac{2xy}{x + y}\right)$ km/hr.

SOLVED EXAMPLES

Ex. 1. ***How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/hr ?*** **(Bank P.O. 2000)**

Sol. Aditya's speed $= 20$ km/hr $= \left(20 \times \frac{5}{18}\right)$ m/sec $= \frac{50}{9}$ m/sec.

$\therefore$ Time taken to cover 400 m $= \left(400 \times \frac{9}{50}\right)$ sec $= 72$ sec $= 1\frac{12}{60}$ min $= 1\frac{1}{5}$ min.

Ex. 2. ***A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist ?*** **(R.R.B. 2002)**

Sol. Speed $= \left(\frac{750}{150}\right)$ m/sec $= 5$m/sec $= \left(5 \times \frac{18}{5}\right)$ km/hr $= 18$ km/hr.

Ex. 3. ***A dog takes 4 leaps for every 5 leaps of a hare but 3 leaps of a dog are equal to 4 leaps of the hare. Compare their speeds.***

Sol. Let the distance covered in 1 leap of the dog be x and that covered in 1 leap of the hare be y.

Then, $3x = 4y \Rightarrow x = \frac{4}{3}y \Rightarrow 4x = \frac{16}{3}y$.

$\therefore$ Ratio of speeds of dog and hare = Ratio of distances covered by them in the same time

$= 4x : 5y = \frac{16}{3}y : 5y = \frac{16}{3} : 5 = 16 : 15.$

Ex. 4. ***While covering a distance of 24 km, a man noticed that after walking for 1 hour and 40 minutes, the distance covered by him was $\frac{5}{7}$ of the remaining distance. What was his speed in metres per second ?*** **(R.R.B. 2002)**

Sol. Let the speed be x km/hr.

Then, distance covered in 1 hr. 40 min. *i.e.*, $1\frac{2}{3}$ hrs $= \frac{5x}{3}$ km.

Remaining distance $= \left(24 - \frac{5x}{3}\right)$ km.

$\therefore \quad \frac{5x}{3} = \frac{5}{7}\left(24 - \frac{5x}{3}\right) \Leftrightarrow \frac{5x}{3} = \frac{5}{7}\left(\frac{72-5x}{3}\right) \Leftrightarrow 7x = 72 - 5x$

$\Leftrightarrow 12x = 72 \quad \Leftrightarrow \quad x = 6$

Hence, speed $= 6$ km/hr $= \left(6 \times \frac{5}{18}\right)$ m/sec $= \frac{5}{3}$ m/sec $= 1\frac{2}{3}$ m/sec.

Ex. 5. ***Peter can cover a certain distance in 1 hr. 24 min. by covering two-third of the distance at 4 kmph and the rest at 5 kmph. Find the total distance.***

Sol. Let the total distance be x km. Then,

$$\frac{\frac{2}{3}x}{4} + \frac{\frac{1}{3}x}{5} = \frac{7}{5} \Leftrightarrow \frac{x}{6} + \frac{x}{15} = \frac{7}{5} \Leftrightarrow 7x = 42 \Leftrightarrow x = 6.$$

$\therefore$ Total distance = 6 km.

Ex. 6. ***A man travelled from the village to the post-office at the rate of 25 kmph and walked back at the rate of 4 kmph. If the whole journey took 5 hours 48 minutes, find the distance of the post-office from the village.*** **(S.S.C. 2004)**

Sol. Average speed $= \left(\frac{2xy}{x+y}\right)$ km/hr $= \left(\frac{2 \times 25 \times 4}{25+4}\right)$ km/hr $= \frac{200}{29}$ km/hr.

Distance travelled in 5 hours 48 minutes *i.e.*, $5\frac{4}{5}$ hrs $= \left(\frac{200}{29} \times \frac{29}{5}\right)$ km = 40 km.

$\therefore$ Distance of the post-office from the village $= \left(\frac{40}{2}\right) = 20$ km.

Ex. 7. ***An aeroplane flies along the four sides of a square at the speeds of 200, 400, 600 and 800 km/hr. Find the average speed of the plane around the field.***

Sol. Let each side of the square be x km and let the average speed of the plane around the field be y km/hr. Then,

$$\frac{x}{200} + \frac{x}{400} + \frac{x}{600} + \frac{x}{800} = \frac{4x}{y} \Leftrightarrow \frac{25x}{2400} = \frac{4x}{y} \Leftrightarrow y = \left(\frac{2400 \times 4}{25}\right) = 384.$$

$\therefore$ Average speed = 384 km/hr.

Ex. 8. ***Walking at $\frac{5}{6}$ of its usual speed, a train is 10 minutes too late. Find its usual time to cover the journey.***

Sol. New speed $= \frac{5}{6}$ of the usual speed

$\therefore$ New time taken $= \frac{6}{5}$ of the usual time

So, $\left(\frac{6}{5} \text{ of the usual time}\right) - (\text{usual time}) = 10$ min.

$\Rightarrow \quad \frac{1}{5}$ of the usual time = 10 min $\quad \Rightarrow \quad$ usual time = 50 min.

18. PROBLEMS ON TRAINS

IMPORTANT FACTS AND FORMULAE

1. $a \text{ km/hr} = \left(a \times \frac{5}{18}\right) \text{m/s}.$ 2. $a \text{ m/s} = \left(a \times \frac{18}{5}\right) \text{km/hr}.$

3. Time taken by a train of length l metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover l metres.

4. Time taken by a train of length l metres to pass a stationary object of length b metres is the time taken by the train to cover $(l + b)$ metres.

5. Suppose two trains or two bodies are moving in the same direction at u m/s and v m/s, where $u > v$, then their relatives speed = $(u - v)$ m/s.

6. Suppose two trains or two bodies are moving in opposite directions at u m/s and v m/s, then their relative speed is = $(u + v)$ m/s.

7. If two trains of length a metres and b metres are moving in opposite directions at u m/s and v m/s, then time taken by the trains to cross each other = $\frac{(a + b)}{(u + v)}$ sec.

8. If two trains of length a metres and b metres are moving in the same direction at u m/s and v m/s, then the time taken by the faster train to cross the slower train = $\frac{(a + b)}{(u - v)}$ sec.

9. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then (A's speed) : (B's speed) = $(\sqrt{b} : \sqrt{a})$.

SOLVED EXAMPLES

Ex. 1. *A train 100 m long is running at the speed of 30 km/hr. Find the time taken by it to pass a man standing near the railway line.* (S.S.C. 2001)

Sol. Speed of the train = $\left(30 \times \frac{5}{18}\right)$ m/sec = $\left(\frac{25}{3}\right)$ m/sec.

Distance moved in passing the standing man = 100 m.

Required time taken = $\frac{100}{\left(\frac{25}{3}\right)} = \left(100 \times \frac{3}{25}\right)$ sec = 12 sec.

Ex. 2. *A train is moving at a speed of 132 km/hr. If the length of the train is 110 metres, how long will it take to cross a railway platform 165 metres long?* (Section-Officers', 2003)

Sol. Speed of train = $\left(132 \times \frac{5}{18}\right)$ m/sec = $\left(\frac{110}{3}\right)$ m/sec.

Distance covered in passing the platform = (110 + 165) m = 275 m.

$\therefore$ Time taken = $\left(275 \times \frac{3}{110}\right)$ sec = $\frac{15}{2}$ sec = $7\frac{1}{2}$ sec.

19. BOATS AND STREAMS

IMPORTANT FACTS AND FORMULAE

1. In water, the direction along the stream is called ***downstream***. And, the direction against the stream is called ***upstream***.
2. If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr, then :

$$\textit{Speed downstream} = (u + v) \text{ km/hr}$$
$$\textit{Speed upstream} = (u - v) \text{ km/hr}.$$

3. If the speed downstream is a km/hr and the speed upstream is b km/hr, then :

$$\textit{Speed in still water} = \frac{1}{2}(a + b) \text{ km/hr}$$
$$\textit{Rate of stream} = \frac{1}{2}(a - b) \text{ km/hr}$$

SOLVED EXAMPLES

Ex. 1. ***A man can row upstream at 7 kmph and downstream at 10 kmph. Find man's rate in still water and the rate of current.***

Sol. Rate in still water $= \frac{1}{2}(10 + 7)$ km/hr $= 8.5$ km/hr.

Rate of current $= \frac{1}{2}(10 - 7)$ km/hr $= 1.5$ km/hr.

Ex. 2. ***A man takes 3 hours 45 minutes to row a boat 15 km downstream of a river and 2 hours 30 minutes to cover a distance of 5 km upstream. Find the speed of the river current in km/hr.***

Sol. Rate downstream $= \left(\dfrac{15}{3\frac{3}{4}}\right)$ km/hr $= \left(15 \times \dfrac{4}{15}\right)$ km/hr $= 4$ km/hr.

Rate upstream $= \left(\dfrac{5}{2\frac{1}{2}}\right)$ km/hr $= \left(5 \times \dfrac{2}{5}\right)$ km/hr $= 2$ km/hr.

$\therefore$ Speed of current $= \frac{1}{2}(4 - 2)$ km/hr $= 1$ km/hr.

Ex. 3. ***A man can row 18 kmph in still water. It takes him thrice as long to row up as to row down the river. Find the rate of stream.***

Sol. Let man's rate upstream be x kmph. Then, his rate downstream $= 3x$ kmph.

$\therefore$ Rate in still water $= \frac{1}{2}(3x + x)$ kmph $= 2x$ kmph.

So, $2x = 18$ or $x = 9$.

$\therefore$ Rate upstream $= 9$ km/hr, Rate downstream $= 27$ km/hr.

Hence, rate of stream $= \frac{1}{2}(27 - 9)$ km/hr $= 9$ km/hr.

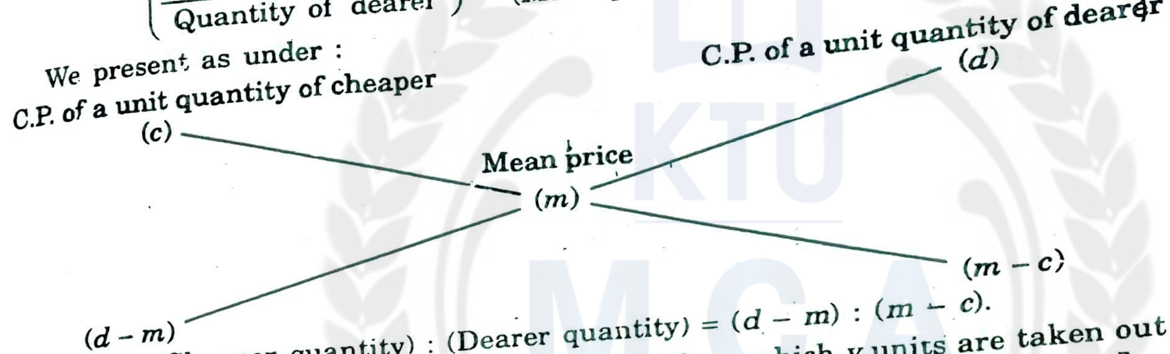
20. ALLIGATION OR MIXTURE

IMPORTANT FACTS AND FORMULAE

1. **Alligation :** It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.
2. **Mean Price :** The cost price of a unit quantity of the mixture is called the mean price.
3. **Rule of Alligation :** If two ingredients are mixed, then

$$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}}\right) = \frac{(\text{C.P. of dearer}) - (\text{Mean price})}{(\text{Mean price}) - (\text{C.P. of cheaper})}.$$

We present as under :



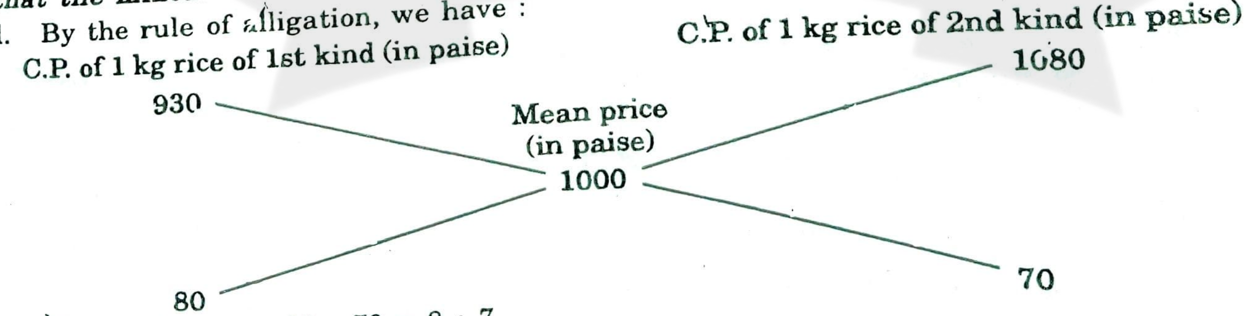
$\therefore$ (Cheaper quantity) : (Dearer quantity) = $(d - m) : (m - c)$.

4. Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid $= \left[x\left(1 - \frac{y}{x}\right)^n\right]$ units.

SOLVED EXAMPLES

Ex. 1. *In what ratio must rice at Rs. 9.30 per kg be mixed with rice at Rs. 10.80 per kg so that the mixture be worth Rs. 10 per kg ?*

Sol. By the rule of alligation, we have :



$\therefore$ Required ratio = 80 : 70 = 8 : 7.

Ex. 2. *How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for Rs. 20 so as to have a mixture worth Rs. $10\frac{2}{3}$ a litre ?*

Sol. C.P. of 1 litre of milk = Rs. $\left(20 \times \frac{2}{3}\right)$ = Rs. $\frac{40}{3}$.

C.P. of 1 litre of water: 0 C.P. of 1 litre of milk: Rs. $\frac{40}{3}$

Mean price $\left(\text{Rs. } \frac{32}{3}\right)$

$\left(\frac{40}{3} - \frac{32}{3}\right) = \frac{8}{3}$ $\left(\frac{32}{3} - 0\right) = \frac{32}{3}$

∴ Ratio of water and milk $= \frac{8}{3} : \frac{32}{3} = 8 : 32 = 1 : 4.$

∴ Quantity of water to be added to 60 litres of milk $= \left(\frac{1}{4} \times 60\right)$ litres = 15 litres.

Ex. 3. ***In what ratio must water be mixed with milk to gain 20% by selling the mixture at cost price ?***

Sol. Let C.P. of milk be Re. 1 per litre.

Then; S.P. of 1 litre of mixture = Re. 1.

Gain obtained = 20%.

∴ C.P. of 1 litre of mixture $= \text{Rs.} \left(\frac{100}{120} \times 1\right) = \text{Re. } \frac{5}{6}.$

By the rule of alligation, we have

C.P. of 1 litre of water: 0 C.P. of 1 litre of milk: Re. 1

$\left(\text{Re. } \frac{5}{6}\right)$

$\left(1 - \frac{5}{6}\right) = \frac{1}{6}$ $\left(\frac{5}{6} - 0\right) = \frac{5}{6}$

∴ Ratio of water and milk $= \frac{1}{6} : \frac{5}{6} = 1 : 5.$

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21. SIMPLE INTEREST

IMPORTANT FACTS AND FORMULAE

1. **Principal** : The money borrowed or lent out for a certain period is called the ***principal*** or the ***sum***.
2. **Interest** : Extra money paid for using other's money is called ***interest***.
3. **Simple Interest (S.I.)** : If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called ***simple interest***.

Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then,

(*i*) $\text{S.I.} = \left(\frac{P \times R \times T}{100}\right)$.

(*ii*) $P = \left(\frac{100 \times \text{S.I.}}{R \times T}\right)$; $R = \left(\frac{100 \times \text{S.I.}}{P \times T}\right)$ and $T = \left(\frac{100 \times \text{S.I.}}{P \times R}\right)$.

SOLVED EXAMPLES

Ex. 1. ***Find the simple interest on Rs. 68,000 at $16\frac{2}{3}$% per annum for 9 months.***

Sol. P = Rs. 68000, $R = \frac{50}{3}\%$ p.a and $T = \frac{9}{12}$ years $= \frac{3}{4}$ years.

$\therefore$ $\text{S.I.} = \left(\frac{P \times R \times T}{100}\right) = \text{Rs.}\left(68000 \times \frac{50}{3} \times \frac{3}{4} \times \frac{1}{100}\right) = \text{Rs. } 8500.$

Ex. 2. ***Find the simple interest on Rs. 3000 at $6\frac{1}{4}$% per annum for the period from 4th Feb., 2005 to 18th April, 2005.***

Sol. Time = (24 + 31 + 18) days = 73 days $= \frac{73}{365}$ year $= \frac{1}{5}$ year.

P = Rs. 3000 and $R = 6\frac{1}{4}\%$ p.a. $= \frac{25}{4}\%$ p.a.

$\therefore$ $\text{S.I.} = \text{Rs.}\left(3000 \times \frac{25}{4} \times \frac{1}{5} \times \frac{1}{100}\right) = \text{Rs. } 37.50.$

Remark : The day on which money is deposited is not counted while the day on which money is withdrawn is counted.

Ex. 3. ***A sum at simple interest at $13\frac{1}{2}$% per annum amounts to Rs. 2502.50 after 4 years. Find the sum.***

Sol. Let sum be Rs. x. Then, $\text{S.I.} = \text{Rs.}\left(x \times \frac{27}{2} \times 4 \times \frac{1}{100}\right) = \text{Rs. } \frac{27x}{50}$.

$\therefore$ Amount $= \text{Rs.}\left(x + \frac{27x}{50}\right) = \text{Rs. } \frac{77x}{50}$.

$\therefore \quad \frac{77x}{50} = 2502.50 \Leftrightarrow x = \frac{2502.50 \times 50}{77} = 1625.$

Hence, sum = Rs. 1625.

Ex. 4. ***A sum of Rs. 800 amounts to Rs. 920 in 3 years at simple interest. If the interest rate is increased by 3%, it would amount to how much ?***

Sol. S.I. = Rs. (920 − 800) = Rs. 120; P = Rs. 800, T = 3 yrs.

$\therefore \quad R = \left(\frac{100 \times 120}{800 \times 3}\right)\% = 5\%.$

New rate = (5 + 3)% = 8%.

New S.I. = Rs. $\left(\frac{800 \times 8 \times 3}{100}\right)$ = Rs. 192.

$\therefore$ New amount = Rs. (800 + 192) = Rs. 992.

Ex. 5. ***Adam borrowed some money at the rate of 6% p.a. for the first two years, at the rate of 9% p.a. for the next three years, and at the rate of 14% p.a. for the period beyond five years. If he pays a total interest of Rs. 11, 400 at the end of nine years, how much money did he borrow ?*** **(Bank P.O. 1999)**

Sol. Let the sum borrowed be x. Then,

$$\left(\frac{x \times 6 \times 2}{100}\right) + \left(\frac{x \times 9 \times 3}{100}\right) + \left(\frac{x \times 14 \times 4}{100}\right) = 11400$$

$$\Leftrightarrow \left(\frac{3x}{25} + \frac{27x}{100} + \frac{14x}{25}\right) = 11400 \Leftrightarrow \frac{95x}{100} = 11400 \Leftrightarrow x = \left(\frac{11400 \times 100}{95}\right) = 12000.$$

Hence, sum borrowed = Rs. 12,000.

Ex. 6. ***A certain sum of money amounts to Rs. 1008 in 2 years and to Rs. 1164 in $3\frac{1}{2}$ years. Find the sum and the rate of interest.***

Sol. S.I. for $1\frac{1}{2}$ years = Rs. (1164 − 1008) = Rs. 156.

S.I. for 2 years = Rs. $\left(156 \times \frac{2}{3} \times 2\right)$ = Rs. 208.

$\therefore$ Principal = Rs. (1008 − 208) = Rs. 800.

Now, P = 800, T = 2 and S.I. = 208.

$\therefore \quad \text{Rate} = \left(\frac{100 \times 208}{800 \times 2}\right)\% = 13\%.$

Ex. 7. ***At what rate percent per annum will a sum of money double in 16 years ?*** **(R.R.B. 2003)**

Sol. Let principal = P. Then, S.I. = P and T = 16 yrs.

$\therefore \quad \text{Rate} = \left(\frac{100 \times P}{P \times 16}\right)\% = 6\frac{1}{4}\%$ p.a.

22. COMPOUND INTEREST

Compound Interest : Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say *yearly* or *half-yearly* or *quarterly* to settle the previous account.

In such cases, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.

After a specified period, *the difference between the amount and the money borrowed is called the* ***Compound Interest*** *(abbreviated as C.I.) for that period.*

IMPORTANT FACTS AND FORMULAE

Let Principal = P, Rate = R% per annum, Time = n years.

I. When interest is compound Annually :

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^n$$

II. When interest is compounded Half-yearly :

$$\text{Amount} = P\left[1 + \frac{(R/2)}{100}\right]^{2n}$$

III. When interest is compounded Quarterly :

$$\text{Amount} = P\left[1 + \frac{(R/4)}{100}\right]^{4n}$$

IV. When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^3 \times \left(1 + \frac{\frac{2}{5}R}{100}\right)$$

V. When Rates are different for different years, say $R_1\%$, $R_2\%$, $R_3\%$ for 1st, 2nd and 3rd year respectively.

$$\text{Then, Amount} = P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right).$$

VI. Present worth of Rs. x due n years hence is given by :

$$\text{Present Worth} = \frac{x}{\left(1 + \frac{R}{100}\right)^n}.$$

SOLVED EXAMPLES

Ex. 1. ***Find compound interest on Rs. 7500 at 4% per annum for 2 years, compounded annually.***

Sol. $$\text{Amount} = \text{Rs.}\left[7500 \times \left(1+\frac{4}{100}\right)^2\right] = \text{Rs.}\left(7500 \times \frac{26}{25} \times \frac{26}{25}\right) = \text{Rs. } 8112.$$

$\therefore$ C.I. = Rs. (8112 − 7500) = Rs. 612.

Ex. 2. ***Find compound interest on Rs. 8000 at 15% per annum for 2 years 4 months, compounded annually.***

Sol. Time = 2 years 4 months = $2\frac{4}{12}$ years = $2\frac{1}{3}$ years.

$$\text{Amount} = \text{Rs.}\left[8000 \times \left(1+\frac{15}{100}\right)^2 \times \left(1+\frac{\frac{1}{3}\times 15}{100}\right)\right] = \text{Rs.}\left(8000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{21}{20}\right)$$

$$= \text{Rs. } 11109.$$

$\therefore$ C.I. = Rs. (11109 − 8000) = Rs. 3109.

Ex. 3. ***Find the compound interest on Rs. 10,000 in 2 years at 4% per annum, the interest being compounded half-yearly.*** (S.S.C. 2000)

Sol. Principal = Rs. 10000; Rate = 2% per half-year; Time = 2 years = 4 half-years.

$$\therefore \quad \text{Amount} = \text{Rs.}\left[10000 \times \left(1+\frac{2}{100}\right)^4\right] = \text{Rs.}\left(10000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50}\right)$$

$$= \text{Rs. } 10824.32.$$

$\therefore$ C.I. = Rs. (10824.32 − 10000) = Rs. 824.32.

Ex. 4. ***Find the compound interest on Rs. 16,000 at 20% per annum for 9 months, compounded quarterly.***

Sol. Principal = Rs. 16000; Time = 9 months = 3 quarters;

Rate = 20% per annum = 5% per quarter.

$$\therefore \quad \text{Amount} = \text{Rs.}\left[16000 \times \left(1+\frac{5}{100}\right)^3\right] = \text{Rs.}\left(16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}\right) = \text{Rs. } 18522.$$

$\therefore$ C.I. = Rs. (18522 − 16000) = Rs. 2522.

Ex. 5. ***If the simple interest on a sum of money at 5% per annum for 3 years is Rs. 1200, find the compound interest on the same sum for the same period at the same rate.***

Sol. Clearly, Rate = 5% p.a., Time = 3 years, S.I. = Rs. 1200.

$$\text{So, Principal} = \text{Rs.}\left(\frac{100 \times 1200}{3 \times 5}\right) = \text{Rs. } 8000.$$

$$\text{Amount} = \text{Rs.}\left[8000 \times \left(1+\frac{5}{100}\right)^3\right] = \text{Rs.}\left(8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}\right) = \text{Rs. } 9261.$$

$\therefore$ C.I. = Rs. (9261 − 8000) = Rs. 1261.

Ex. 6. ***In what time will Rs. 1000 become Rs. 1331 at 10% per annum compounded***

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