12. RATIO AND PROPORTION

IMPORTANT FACTS AND FORMULAE

I. **RATIO** : The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.

In the ratio $a : b$, we call a as the **first term** or **antecedent** and b, the **second term** or **consequent**.

Ex. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule : The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Ex. 4 : 5 = 8 : 10 = 12 : 15 etc. Also, 4 : 6 = 2 : 3.

2. **PROPORTION** : *The equality of two ratios is called proportion.*

If $a : b = c : d$, we write, $a : b :: c : d$ and we say that a, b, c, d are in proportion. Here a and d are called **extremes**, while b and c are called **mean terms**.

Product of means = Product of extremes.

Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. (*i*) **Fourth Proportional** : If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

(*ii*) **Third Proportional** : If $a : b = b : c$, then c is called the third proportional to a and b.

(*iii*) **Mean Proportional** : Mean proportional between a and b is $\sqrt{ab}$.

4. (*i*) COMPARISON OF RATIOS :

We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

(*ii*) COMPOUNDED RATIO :

The compounded ratio of the ratios $(a : b), (c : d), (e : f)$ is $(ace : bdf)$.

5. (*i*) ***Duplicate ratio*** of $(a : b)$ is $(a^2 : b^2)$.

(*ii*) ***Sub-duplicate ratio*** of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

(*iii*) ***Triplicate ratio*** of $(a : b)$ is $(a^3 : b^3)$.

(*iv*) **Sub-triplicate ratio** of $(a : b)$ is $\left(a^{\frac{1}{3}} : b^{\frac{1}{3}}\right)$.

(*v*) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. **(componendo and dividendo)**

6. VARIATION :

(*i*) We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$.

(*ii*) We say that x is inversely proportional to y, if $xy = k$ for some constant k and we write, $x \propto \frac{1}{y}$.

SOLVED PROBLEMS

Ex. 1. ***If a : b = 5 : 9 and b : c = 4 : 7, find a : b : c.***

Sol. $a : b = 5 : 9$ and $b : c = 4 : 7 = \left(4 \times \frac{9}{4}\right) : \left(7 \times \frac{9}{4}\right) = 9 : \frac{63}{4}$

$\Rightarrow$ $a : b : c = 5 : 9 : \frac{63}{4} = 20 : 36 : 63.$

Ex. 2. ***Find :***

(i) the fourth proportional to 4, 9, 12;

(ii) the third proportional to 16 and 36;

(iii) the mean proportional between 0.08 and 0.18.

Sol. (i) Let the fourth proportional to 4, 9, 12 be x.

Then, $4 : 9 :: 12 : x \Leftrightarrow 4 \times x = 9 \times 12 \Leftrightarrow x = \frac{9 \times 12}{4} = 27.$

$\therefore$ Fourth proportional to 4, 9, 12 is 27.

(ii) Let the third proportional to 16 and 36 be x.

Then, $16 : 36 :: 36 : x \Leftrightarrow 16 \times x = 36 \times 36 \Leftrightarrow x = \frac{36 \times 36}{16} = 81.$

$\therefore$ Third proportional to 16 and 36 is 81.

(iii) Mean proportional between 0.08 and 0.18

$= \sqrt{0.08 \times 0.18} = \sqrt{\frac{8}{100} \times \frac{18}{100}} = \sqrt{\frac{144}{100 \times 100}} = \frac{12}{100} = 0.12.$

Ex. 3. ***If x : y = 3 : 4, find (4x + 5y) : (5x − 2y).***

Sol. $\frac{x}{y} = \frac{3}{4} \Rightarrow \frac{4x + 5y}{5x - 2y} = \frac{4\left(\frac{x}{y}\right) + 5}{5\left(\frac{x}{y}\right) - 2} = \frac{\left(4 \times \frac{3}{4} + 5\right)}{\left(5 \times \frac{3}{4} - 2\right)} = \frac{(3 + 5)}{\left(\frac{7}{4}\right)} = \frac{32}{7}.$

Ex. 4. ***Divide Rs. 672 in the ratio 5 : 3.***

Sol. Sum of ratio terms = (5 + 3) = 8.

$\therefore$ First part = Rs. $\left(672 \times \frac{5}{8}\right)$ = Rs. 420; Second part = Rs. $\left(672 \times \frac{3}{8}\right)$ = Rs. 252.

Ex. 5. ***Divide Rs. 1162 among A, B, C in the ratio 35 : 28 : 20.***

Sol. Sum of ratio terms = (35 + 28 + 20) = 83.

A's share = Rs. $\left(1162 \times \frac{35}{83}\right)$ = Rs. 490; B's share = Rs. $\left(1162 \times \frac{28}{83}\right)$ = Rs. 392;

C's share = Rs. $\left(1162 \times \frac{20}{83}\right)$ = Rs. 280.

Ex. 6. ***A bag contains 50 p, 25 p and 10 p coins in the ratio 5 : 9 : 4, amounting to Rs. 206. Find the number of coins of each type.***

Sol. Let the number of 50 p, 25 p and 10 p coins be $5x$, $9x$ and $4x$ respectively.

Then, $\frac{5x}{2} + \frac{9x}{4} + \frac{4x}{10} = 206$

$\Leftrightarrow$ $50x + 45x + 8x = 4120 \Leftrightarrow 103x = 4120 \Leftrightarrow x = 40.$

$\therefore$ Number of 50 p coins = (5 × 40) = 200; Number of 25 p coins = (9 × 40) = 360;

Number of 10 p coins = (4 × 40) = 160.

13. PARTNERSHIP

IMPORTANT FACTS AND FORMULAE

1. **Partnership :** When two or more than two persons run a business jointly, they are called ***partners*** and the deal is known as ***partnership.***

2. **Ratio of Division of Gains :**

   (i) *When investments of all the partners are for the same time*, the gain or loss is distributed among the partners in the ratio of their investments.

   Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year :

   (A's share of profit) : (B's share of profit) = $x : y$.

   (ii) *When investments are for different time periods*, then equivalent capitals are calculated for a unit of time by taking (capital × number of units of time). Now, gain or loss is divided in the ratio of these capitals.

   Suppose A invests Rs. x for p months and B invests Rs. y for q months, then

   (A's share of profit) : (B's share of profit) = $xp : yq$.

3. **Working and Sleeping Partners :** A partner who manages the business is known as a ***working partner*** and the one who simply invests the money is a ***sleeping partner.***

SOLVED EXAMPLES

Ex. 1. *A, B and C started a business by investing Rs. 1,20,000, Rs. 1,35,000 and Rs. 1,50,000 respectively. Find the share of each, out of an annual profit of Rs. 56,700.*

Sol. Ratio of shares of A, B and C = Ratio of their investments

= 120000 : 135000 : 150000 = 8 : 9 : 10.

∴ A's share = Rs. $\left(56700 \times \frac{8}{27}\right)$ = Rs. 16800.

B's share = Rs. $\left(56700 \times \frac{9}{27}\right)$ = Rs. 18900.

C's share = Rs. $\left(56700 \times \frac{10}{27}\right)$ = Rs. 21000.

Ex. 2. *Alfred started a business investing Rs. 45,000. After 3 months, Peter joined him with a capital of Rs. 60,000. After another 6 months, Ronald joined them with a capital of Rs. 90,000. At the end of the year, they made a profit of Rs. 16,500. Find the share of each.*

Sol. Clearly, Alfred invested his capital for 12 months, Peter for 9 months and Ronald for 3 months.

So, ratio of their capitals = (45000 × 12) : (60000 × 9) : (90000 × 3)

= 540000 : 540000 : 270000 = 2 : 2 : 1.

∴ Alfred's share = Rs. $\left(16500 \times \frac{2}{5}\right)$ = Rs. 6600;

Peter's share $= \text{Rs.}\left(16500 \times \frac{2}{5}\right) = \text{Rs. } 6600;$

Ronald's share $= \text{Rs.}\left(16500 \times \frac{1}{5}\right) = \text{Rs. } 3300.$

Ex. 3. ***A, B and C start a business each investing Rs. 20,000. After 5 months A withdrew Rs. 5000, B withdrew Rs. 4000 and C invests Rs. 6000 more. At the end of the year, a total profit of Rs. 69,900 was recorded. Find the share of each.***

Sol. Ratio of the capitals of A, B and C

$$= 20000 \times 5 + 15000 \times 7 : 20000 \times 5 + 16000 \times 7 : 20000 \times 5 + 26000 \times 7$$

$$= 205000 : 212000 : 282000 = 205 : 212 : 282.$$

$\therefore$ A's share $= \text{Rs.}\left(69900 \times \frac{205}{699}\right) = \text{Rs. } 20500;$

B's share $= \text{Rs.}\left(69900 \times \frac{212}{699}\right) = \text{Rs. } 21200;$

C's share $= \text{Rs.}\left(69900 \times \frac{282}{699}\right) = \text{Rs. } 28200.$

Ex. 4. ***A, B and C enter into partnership. A invests 3 times as much as B invests and B invests two-third of what C invests. At the end of the year, the profit earned is Rs. 6600. What is the share of B ?***

Sol. Let C's capital = Rs. x. Then, B's capital = Rs. $\frac{2}{3}x$.

A's capital $= \text{Rs.}\left(3 \times \frac{2}{3}x\right) = \text{Rs. } 2x.$

$\therefore$ Ratio of their capitals $= 2x : \frac{2}{3}x : x = 6 : 2 : 3.$

Hence, B's share $= \text{Rs.}\left(6600 \times \frac{2}{11}\right) = \text{Rs. } 1200.$

Ex. 5. ***Four milkmen rented a pasture. A grazed 24 cows for 3 months; B 10 cows for 5 months; C 35 cows for 4 months and D 21 cows for 3 months. If A's share of rent is Rs. 720, find the total rent of the field.***

Sol. Ratio of shares of A, B, C, D $= (24 \times 3) : (10 \times 5) : (35 \times 4) : (21 \times 3)$

$$= 72 : 50 : 140 : 63.$$

Let total rent be Rs. x. Then, A's share = Rs. $\frac{72x}{325}$.

$$\therefore \quad \frac{72x}{325} = 720 \Leftrightarrow x = \frac{720 \times 325}{72} = 3250.$$

Hence, total rent of the field is Rs. 3250.

Ex. 6. ***A invested Rs. 76,000 in a business. After few months, B joined him with Rs. 57,000. At the end of the year, the total profit was divided between them in the ratio 2 : 1. After how many months did B join ?***

Sol. Suppose B joined after x months. Then, B's money was invested for $(12 - x)$ months.

$$\therefore \quad \frac{76000 \times 12}{57000 \times (12 - x)} = \frac{2}{1} \Leftrightarrow 912000 = 114000\ (12 - x)$$

$$\Leftrightarrow 114\ (12 - x) = 912 \Leftrightarrow (12 - x) = 8 \Leftrightarrow x = 4.$$

Hence, B joined after 4 months.

14. CHAIN RULE

IMPORTANT FACTS AND FORMULAE

1. **Direct Proportion** : *Two quantities are said to be directly proportional, if on the increase (or decrease) of the one, the other increases (or decreases) to the same extent.*

   Ex. 1. ***Cost is directly proportional to the number of articles.***
   (More Articles, More Cost)

   Ex. 2. ***Work done is directly proportional to the number of men working on it.***
   (More Men, More Work)

2. **Indirect Proportion** : *Two quantities are said to be indirectly proportional, if on the increase of the one, the other decreases to the same extent and vice-versa.*

   Ex. 1. ***The time taken by a car in covering a certain distance is inversely proportional to the speed of the car.***
   (More speed, Less is the time taken to cover a distance)

   Ex. 2. ***Time taken to finish a work is inversely proportional to the number of persons working at it.***
   (More persons, Less is the time taken to finish a job)

   Remark : In solving questions by chain rule, we compare every item with the term to be found out.

SOLVED EXAMPLES

Ex. 1. ***If 15 toys cost Rs. 234, what do 35 toys cost ?***

Sol. Let the required cost be Rs. x. Then,

More toys, More cost *(Direct Proportion)*

$$\therefore \quad 15 : 35 :: 234 : x \Leftrightarrow (15 \times x) = (35 \times 234) \Leftrightarrow x = \left(\frac{35 \times 234}{15}\right) = 546.$$

Hence, the cost of 35 toys is Rs. 546.

Ex. 2. ***If 36 men can do a piece of work in 25 hours, in how many hours will 15 men do it ?***

Sol. Let the required number of hours be x. Then,

Less men, More hours *(Indirect Proportion)*

$$\therefore \quad 15 : 36 :: 25 : x \Leftrightarrow (15 \times x) = (36 \times 25) \Leftrightarrow x = \frac{36 \times 25}{15} = 60.$$

Hence, 15 men can do it in 60 hours.

Ex. 3. ***If the wages of 6 men for 15 days be Rs. 2100, then find the wages of 9 men for 12 days.***

Sol. Let the required wages be Rs. x.

More men, More wages *(Direct Proportion)*

Less days, Less wages *(Direct Proportion)*

$$\left.\begin{array}{ll} \text{Men} & 6 : 9 \\ \text{Days} & 15 : 12 \end{array}\right\} :: 2100 : x$$

$\therefore$ $(6 \times 15 \times x) = (9 \times 12 \times 2100) \Leftrightarrow x = \left(\frac{9 \times 12 \times 2100}{6 \times 15}\right) = 2520.$

Hence, the required wages are Rs. 2520.

Ex. 4. ***If 20 men can build a wall 56 metres long in 6 days, what length of a similar wall can be built by 35 men in 3 days ?***

Sol. Let the required length be x metres.

More men, More length built (*Direct Proportion*)

Less days, Less length built (*Direct Proportion*)

$$\left.\begin{array}{ll}\text{Men} & 20 : 35 \\ \text{Days} & 6 : 3\end{array}\right\} :: 56 : x$$

$\therefore$ $(20 \times 6 \times x) = (35 \times 3 \times 56) \Leftrightarrow x = \frac{(35 \times 3 \times 56)}{120} = 49.$

Hence, the required length is 49 m.

Ex. 5. ***If 15 men, working 9 hours a day, can reap a field in 16 days, in how many days will 18 men reap the field, working 8 hours a day ?***

Sol. Let the required number of days be x.

More men, Less days (*Indirect Proportion*)

Less hours per day, More days (*Indirect Proportion*)

$$\left.\begin{array}{ll}\text{Men} & 18 : 15 \\ \text{Hours per day} & 8 : 9\end{array}\right\} :: 16 : x$$

$\therefore$ $(18 \times 8 \times x) = (15 \times 9 \times 16) \Leftrightarrow x = \left(\frac{15 \times 144}{144}\right) = 15.$

Hence, required number of days = 15.

Ex. 6. ***If 9 engines consume 24 metric tonnes of coal, when each is working 8 hours a day, how much coal will be required for 8 engines, each running 13 hours a day, it being given that 3 engines of former type consume as much as 4 engines of latter type?***

Sol. Let 3 engines of former type consume 1 unit in 1 hour.

Then, 4 engines of latter type consume 1 unit in 1 hour.

$\therefore$ 1 engine of former type consumes $\frac{1}{3}$ unit in 1 hour.

1 engine of latter type consumes $\frac{1}{4}$ unit in 1 hour.

Let the required consumption of coal be x units.

Less engines, Less coal consumed (*Direct Proportion*)

More working hours, More coal consumed (*Direct Proportion*)

Less rate of consumption, Less coal consumed (*Direct Proportion*)

$$\left.\begin{array}{ll}\text{Number of engines} & 9 : 8 \\ \text{Working hours} & 8 : 13 \\ \text{Rate of consumption} & \frac{1}{3} : \frac{1}{4}\end{array}\right\} :: 24 : x$$

$\therefore$ $\left(9 \times 8 \times \frac{1}{3} \times x\right) = \left(8 \times 13 \times \frac{1}{4} \times 24\right) \Leftrightarrow 24x = 624 \Leftrightarrow x = 26.$

Hence, the required consumption of coal = 26 metric tonnes.

15. TIME AND WORK

IMPORTANT FACTS AND FORMULAE

1. If A can do a piece of work in n days, then A's 1 day's work $= \frac{1}{n}$.
2. If A's 1 day's work $= \frac{1}{n}$, then A can finish the work in n days.
3. If A is thrice as good a workman as B, then :
   Ratio of work done by A and B = 3 : 1.
   Ratio of times taken by A and B to finish a work = 1 : 3.

SOLVED EXAMPLES

Ex. 1. ***Worker A takes 8 hours to do a job. Worker B takes 10 hours to do the same job. How long should it take both A and B, working together but independently, to do the same job ?*** **(IGNOU, 2003)**

Sol. A's 1 hour's work $= \frac{1}{8}$, B's 1 hour's work $= \frac{1}{10}$.

(A + B)'s 1 hour's work $= \left(\frac{1}{8}+\frac{1}{10}\right) = \frac{9}{40}$.

$\therefore$ Both A and B will finish the work in $\frac{40}{9} = 4\frac{4}{9}$ days.

Ex. 2. ***A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work?*** **(Bank P.O. 2003)**

Sol. (A + B)'s 1 day's work $= \frac{1}{4}$, A's 1 day's work $= \frac{1}{12}$.

$\therefore$ B's 1 day's work $= \left(\frac{1}{4}-\frac{1}{12}\right) = \frac{1}{6}$.

Hence, B alone can complete the work in 6 days.

Ex. 3. ***A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days of 7 hours each. How long will they take to do it, working together $8\frac{2}{5}$ hours a day?***

Sol. A can complete the work in (7 × 9) = 63 hours.
B can complete the work in (6 × 7) = 42 hours.

$\therefore$ A's 1 hour's work $= \frac{1}{63}$ and B's 1 hour's work $= \frac{1}{42}$.

(A + B)'s 1 hour's work $= \left(\frac{1}{63}+\frac{1}{42}\right) = \frac{5}{126}$.

$\therefore$ Both will finish the work in $\left(\frac{126}{5}\right)$ hrs.

Number of days of $8\frac{2}{5}$ hrs each $= \left(\frac{126}{5}\times\frac{5}{42}\right) = 3$ days.

16. PIPES AND CISTERNS

IMPORTANT FACTS AND FORMULAE

1. **Inlet** : A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

   Outlet : A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

2. (*i*) If a pipe can fill a tank in x hours, then :

   part filled in 1 hour $= \frac{1}{x}$.

   (*ii*) If a pipe can empty a full tank in y hours, then :

   part emptied in 1 hour $= \frac{1}{y}$.

   (*iii*) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$), then on opening both the pipes, the net part filled in 1 hour $= \left(\frac{1}{x} - \frac{1}{y}\right)$.

   (*iv*) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $x > y$), then on opening both the pipes, the net part emptied in 1 hour $= \left(\frac{1}{y} - \frac{1}{x}\right)$.

SOLVED EXAMPLES

Ex. 1. *Two pipes A and B can fill a tank in 36 hours and 45 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?*

Sol. Part filled by A in 1 hour $= \frac{1}{36}$; Part filled by B in 1 hour $= \frac{1}{45}$.

Part filled by (A + B) in 1 hour $= \left(\frac{1}{36} + \frac{1}{45}\right) = \frac{9}{180} = \frac{1}{20}$.

Hence, both the pipes together will fill the tank in 20 hours.

Ex. 2. *Two pipes can fill a tank in 10 hours and 12 hours respectively while a third pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time will the tank be filled?*

Sol. Net part filled in 1 hour $= \left(\frac{1}{10} + \frac{1}{12} - \frac{1}{20}\right) = \frac{8}{60} = \frac{2}{15}$.

$\therefore$ The tank will be full in $\frac{15}{2}$ hrs = 7 hrs 30 min.

Ex. 3. *If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does it take the second pipe to fill the reservoir?*

Sol. Let the reservoir be filled by first pipe in x hours.

Then, second pipe will fill it in $(x + 10)$ hours.

$$\therefore \quad \frac{1}{x} + \frac{1}{(x+10)} = \frac{1}{12} \quad \Leftrightarrow \quad \frac{x+10+x}{x(x+10)} = \frac{1}{12}$$

$$\Leftrightarrow \quad x^2 - 14x - 120 = 0 \quad \Leftrightarrow \quad (x-20)(x+6) = 0$$

$$\Leftrightarrow \quad x = 20 \qquad \text{[neglecting the – ve value of } x\text{]}$$

So, the second pipe will take $(20 + 10)$ hrs i.e. 30 hrs to fill the reservoir.

Ex. 4. ***A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the three are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?***

Sol. Work done by the waste pipe in 1 minute

$$= \frac{1}{20} - \left(\frac{1}{12} + \frac{1}{15}\right) = -\frac{1}{10} \qquad \text{[– ve sign means emptying]}$$

$\therefore$ Waste pipe will empty the full cistern in 10 minutes.

Ex. 5. ***An electric pump can fill a tank in 3 hours. Because of a leak in the tank it took $3\frac{1}{2}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?***

Sol. Work done by the leak in 1 hour $= \left[\frac{1}{3} - \frac{1}{\left(\frac{7}{2}\right)}\right] = \left(\frac{1}{3} - \frac{2}{7}\right) = \frac{1}{21}$

$\therefore$ The leak will empty the tank in 21 hours.

Ex. 6. ***Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?***

Sol. Work done by the two pipes in 1 hour $= \left(\frac{1}{14} + \frac{1}{16}\right) = \frac{15}{112}$

$\therefore$ Time taken by these pipes to fill the tank $= \frac{112}{15}$ hrs $= 7$ hrs 28 min.

Due to leakage, time taken $= 7$ hrs 28 min $+ 32$ min $= 8$ hrs

$\therefore$ Work done by (two pipes + leak) in 1 hour $= \frac{1}{8}$

Work done by the leak in 1 hour $= \left(\frac{15}{112} - \frac{1}{8}\right) = \frac{1}{112}$

$\therefore$ Leak will empty the full cistern in 112 hours.

Ex. 7. ***Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A water pipe C can empty the tank in 30 min. First A and B are opened. After 7 minutes, C is also opened. In how much time, the tank is full?***

Sol. Part filled in 7 min. $= 7\left(\frac{1}{36} + \frac{1}{45}\right) = \frac{7}{20}$

Remaining part $= \left(1 - \frac{7}{20}\right) = \frac{13}{20}$

Net part filled in 1 min. when A, B and C are opened $= \left(\frac{1}{36} + \frac{1}{45} - \frac{1}{30}\right) = \frac{1}{60}$.

Now, $\frac{1}{60}$ part is filled in 1 min.

$\frac{13}{20}$ part is filled in $\left(60 \times \frac{13}{20}\right) = 39$ min.

Total time taken to fill the tank = (39 + 7) min. = 46 min.

Ex. 8. ***Two pipes A and B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 minutes ?***

Sol. Let B be closed after x minutes. Then,

part filled by (A + B) in x min. + part filled by A in $(18 - x)$ min. = 1

$\therefore \quad x\left(\frac{1}{24} + \frac{1}{32}\right) + (18 - x) \times \frac{1}{24} = 1 \quad \Leftrightarrow \quad \frac{7x}{96} + \frac{18 - x}{24} = 1$

$\Leftrightarrow \quad 7x + 4(18 - x) = 96 \quad \Leftrightarrow \quad x = 8.$

Hence, B must be closed after 8 minutes.