SECTION-I ARITHMETICAL ABILITY

1. OPERATIONS ON NUMBERS

1. **NUMBERS:** In Hindu- Arabic system, we have ten **digits**, namely 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called zero, one two, three, four, five, six, seven, eight and nine respectively.

A number is denoted by a group of digits, called **numeral.**

For denoting a numeral, we use the place-value chart, given below.

Ex. 1. *Write each of the following numerals in words.*

	Ten-Crores	*Crores*	*Ten-Lakhs*	*Lakhs*	*Ten-Thousands*	*Thousands*	*Hundreds*	*Tens*	*Units*
(i)				6	3	8	5	4	9
(ii)			2	3	8	0	9	1	7
(iii)		8	5	4	1	6	0	0	8
(iv)	5	6	1	3	0	7	0	9	0

Sol. The given numerals in words are:

(*i*) Six lakh thirty-eight thousand five hundred forty-nine.

(*ii*) Twenty-three lakh eighty thousand nine hundred seventeen.

(*iii*) Eight crore fifty-four lakh sixteen thousand eight.

(*iv*) Fifty-six crore thirteen lakh seven thousand ninety.

Ex. 2. *Write each of the following numbers in figures:*

(*i*) Nine crore four lakh six thousand two

(*ii*) Twelve crore seven lakh nine thousand two hundred seven.

(*iii*) Four lakh four thousand forty.

(*iv*) Twenty-one crore sixty lakh five thousand fourteen.

Sol. Using the place value chart, we may write

	Ten-Crores	*Crores*	*Ten-Lakhs*	*Lakhs*	*Ten-Thousands*	*Thousands*	*Hundreds*	*Tens*	*Units*
(i)		9	0	4	0	6	0	0	2
(ii)	1	2	0	7	0	9	2	0	7
(iii)				4	0	4	0	4	0
(iv)	2	1	6	0	0	5	0	1	4

2. Face value and Place value (or Local Value) of a Digit In a Numeral

(*i*) The face value of a digit in a numeral is its own value, at whatever place it may be

Ex. In the numeral 6872, the face value of 2 is 2, the face value of 7 is 7, the face value of 8 is 8 and the face value of 6 is 6.

(*ii*) In a given numeral:

Place value of unit digit = (unit digit) × 1,

Place value of tens digit = (tens digit) × 10,

Place value of hundreds digit = (hundreds digit) × 100 and so on.

Ex. In the numeral 70984, we have

Place value of 4 = (4 × 1) = 4

Place value of 8 = (8 × 10) = 80,

Place value of 9 = (9 × 100) = 900,

Place value of 7 = (7 × 10000) = 70000.

Note: Place value of 0 in a given numeral is 0, at whatever place it may be.

Ex. 3. *In the numeral 8734925, write down:*

(*i*) *Face value of 7* (*ii*) *Face value of 9* (*iii*) *Place value of 4*

(*iv*) *Place value of 3* (*iv*) *Place value of 8* (*v*) *Place value of 5*

Sol. Writing the given numeral in place-value chart, we get

Ten-Lakhs	Lakhs	Ten-thousands	Thousands	Hundreds	Tens	Ones
8	7	3	4	9	9	5

(*i*) Face value of 7 is 7.

(*ii*) Face value of 9 is 9.

(*iii*) Place value of 4 = (4 × 1000) = 4000.

(*iv*) Place value of 3 = (3 × 10000) = 30000.

(*v*) Place value of 8 = (8 × 1000000) = 8000000.

(*vi*) Place value of 5 = (5 × 1) = 5.

3. Various Types of Numbers:

(*i*) **Natural Numbers:** *Counting numbers are called natural numbers.*

Thus 1, 2, 3, 4, 5, 6, ... are all natural numbers.

(*ii*) **Whole Numbers:** *All counting numbers and 0 form the set of whole numbers.*

Thus 0, 1, 2, 3, 4, 5, ... etc. are whole numbers.

Clearly, every natural number is a whole number and 0 is a whole number which is not a natural number.

(*iii*) **Integers:** All counting numbers, zero and negatives of counting numbers form the set of integers.

Thus, ..., –3, –2, –1, 0, 1 , 2, 3, ... are all integers.

Set of positive integers = {1, 2, 3, 4, 5, 6, ...}

Set of negative integers = {–1, –2, –3, –4, ...}

Set of all non-negative integers = {0, 1, 2, 3, 4, 5, ...}.

4. Even And Odd Numbers:

(*i*) **Even Numbers:** *A counting number divisible by 2 is called an even number.*

Thus 0, 2, 4, 6, 8, 10, 12, ... etc. are all even numbers.

(*ii*) **Odd Numbers:** *A counting number not divisible by 2 is called an odd number.*

Thus 1, 3, 5, 7, 9, 11, 13, 15, ... etc. are all odd numbers.

5. Prime Numbers: *A counting number is called a prime number if it has exactly two factors, namely itself and 1.*

Ex. All prime numbers less than 100 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Test For a Number To be Prime:

Let p be a given number and let n be the smallest counting number such that $n^2 \geq p$.

Now, test whether p is divisible by any of the prime numbers less than or equal to n. If yes, then p is not prime otherwise, p is prime.

Ex. 4. *Test, which of the following are prime numbers ?*

(*i*) 137 (*ii*) 173 (*iii*) 319 (*iv*) 437 (*v*) 811

Sol. (*i*) We know that $(12)^2 > 137$.

Prime numbers less than 12 are 2, 3, 5, 7, 11.

Clearly, none of them divides 137.

$\therefore$ 137 is a prime number.

(*ii*) We known that $(14)^2 > 173$.

Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.

Clearly, none of them divides 173.

$\therefore$ 173 is a prime number.

(*iii*) We know that $(18)^2 > 319$.

Prime numbers less than 18 are 2, 3, 5, 7, 11, 13, 17.

Out of these prime numbers, 11 divides 319 completely.

$\therefore$ 319 is not a prime number.

(*iv*) We know that $(21)^2 > 437$.

Prime numbers less than 21 are 2, 3, 5, 7, 11, 13, 17, 19.

Clearly, 437 is divisible by 19.

$\therefore$ 437 is not a prime number.

(*v*) We know that $(29)^2 > 811$.

Prime numbers less than 29 are 2, 3, 5, 7, 11, 13, 17, 19, 23.

Clearly, none of these numbers divides 811.

$\therefore$ 811 is a prime number.

Composite Numbers: *The natural numbers which are not prime, are called composite numbers.*

6. Co Primes: Two natural numbers a and b are said to be co-prime if their HCF is 1.

Ex. (2, 3), (4, 5), (7, 9), (8, 11) etc. are pairs of co-primes.

TESTS OF DIVISIBILITY

I. Divisibility By 2:

A number is divisible by 2 if its unit digit is any of 0, 2, 4, 6, 8.

Ex. 58694 is divisible by 2, while 86945 is not divisible by 2.

II. Divisibility By 3:

A number is divisible by 3 only when the sum of its digits is divisible by 3.

Ex. (*i*) In the number 695421, the sum of digits = 27, which is divisible by 3.

$\therefore$ 695421 is divisible by 3.

(*ii*) In the number 948653, the sum of digits = 35, which is not divisible by 3.

$\therefore$ 948653 is not divisible by 3.

III. Divisibility By 9:

A number is divisible by 9 only when the sum of its digits is divisible by 9.

Ex. (*i*) In the number 246591, the sum of digits = 27, which is divisible by 9.

$\therefore$ 246591 is divisible by 9.

(*ii*) In the number 734519, the sum of digits = 29, which is not divisible by 9.

$\therefore$ 734519 is not divisible by 9.

IV. Divisibility By 4:

A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

Ex. (*i*) 6879376 is divisible by 4, since 76 is divisible by 4.

(*ii*) 496138 is not divisible by 4, since 38 is not divisible by 4.

V. Divisibility By 8:

A number is divisible by 8 if the number formed by hundred's, ten's and unit's digit of the given number is divisible by 8.

Ex. (*i*) In the number 16789352, the number formed by last 3 digits, namely 352 is divisible by 8.

∴ 16789352 is divisible by 8.

(*ii*) In the number 576484, the number formed by last 3 digits, namely 484 is not divisible by 8.

∴ 576484 is not divisible by 8.

VI. Divisibility By 10:

A number is divisible by 10 only when its unit digit is 0.

Ex. (*i*) 7849320 is divisible by 10, since its unit digit is 0.

(*ii*) 678405 is not divisible by 10, since its unit digit is not 0.

VII. Divisibility By 5:

A number is divisible by 5 only when its unit digit is 0 or 5.

Ex. Each of the numbers 76895 and 68790 is divisible by 5.

VIII. Divisibility By 11:

A number is divisible by 11 if the difference between the sum of its digits at odd places and the sum of its digits at even places is either 0 or a number divisible by 11.

Ex. (*i*) Consider the number 29435417.

(Sum of its digits at odd places) – (Sum of its digits at even places)

= (7 + 4 + 3 + 9) – (1 + 5 + 4 + 2) = (23 – 12) = 11, which is divisible by 11.

∴ 29435417 is divisible by 11.

(*ii*) Consider the number 57463822.

(Sum of its digits at odd places) – (Sum of its digits at even places)

= (2 + 8 + 6 + 7) – (2 + 3 + 4 + 5) = (23 – 14) = 9, which is not divisible by 11.

∴ 57463822 is not divisible by 11.

SOLVED EXAMPLES

Ex. 1. *9587 – ? = 7429 – 4358.*

Sol. Let 9587 – x = 7429 – 4358. Then,

9587 – x = 3071 ⇒ x = 9587 – 3071 = 6516.

7429
– 4358
3071

Ex. 2. *5793405 × 9999 = ?*

Sol. 5793405 × 9999 = 5793405 × (10000 – 1)

= 57934050000 – 5793405

= 57928256595

57934050000
–5793405
57928256595

Ex. 3. *839478 × 625 = ?*

Sol. 839478 × 625 = 839478 × 5^4

$$= 839478 \times \left(\frac{10}{2}\right)^4 = \frac{839478 \times 10^4}{2^4}$$

$$= \frac{8394780000}{16} = 524673750.$$

2. H.C.F. AND L.C.M. OF NUMBERS

IMPORTANT FACTS AND FORMULAE

I. **Factors and Multiples** : If a number a divides another number b exactly, we say that a is a ***factor*** of b. In this case, b is called a ***multiple*** of a.

II. **Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.)** : The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are two methods of finding the H.C.F. of a given set of numbers :

1. ***Factorization Method*** : *Express each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.*
2. ***Division Method*** : *Suppose we have to find the H.C.F. of two given numbers. Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.*

Finding the H.C.F. of more than two numbers : Suppose we have to find the H.C.F. of three numbers. Then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers.

Similarly, the H.C.F. of more than three numbers may be obtained.

III. **Least Common Multiple (L.C.M.)** : The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

1. ***Factorization Method of Finding L.C.M.*** : *Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.*
2. ***Common Division Method (Short-cut Method) of Finding L.C.M.*** : *Arrange the given numbers in a row in any order. Divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.*

IV. **Product of two numbers = Product of their H.C.F. and L.C.M.**

V. **Co-primes** : Two numbers are said to be co-primes if their H.C.F. is 1.

VI. **H.C.F. and L.C.M. of Fractions** :

1. $\text{H.C.F.} = \dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$ 2. $\text{L.C.M.} = \dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$.

VII. **H.C.F. and L.C.M. of Decimal Fractions** : *In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.*

VIII. **Comparison of Fractions** : *Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.*

SOLVED EXAMPLES

Ex. 1. ***Find the H.C.F. of*** $2^3 \times 3^2 \times 5 \times 7^4$, $2^2 \times 3^5 \times 5^2 \times 7^3$, $2^3 \times 5^3 \times 7^2$.

Sol. The prime numbers common to given numbers are 2, 5 and 7

∴ H.C.F. = $2^2 \times 5 \times 7^2 = 980$.

Ex. 2. ***Find the H.C.F. of 108, 288 and 360.***

Sol. $108 = 2^2 \times 3^3$, $288 = 2^5 \times 3^2$ and $360 = 2^3 \times 5 \times 3^2$.

∴ H.C.F. = $2^2 \times 3^2 = 36$.

Ex. 3. ***Find the H.C.F. of 513, 1134 and 1215.***

Sol.

```
1134 ) 1215 ( 1
       1134
       ----
         81 ) 1134 ( 14
                81
               ---
               324
               324
               ---
                 ×
```

∴ H.C.F. of 1134 and 1215 is 81.

So, Required H.C.F. = H.C.F. of 513 and 81.

```
81 ) 513 ( 6
     486
     ---
      27 ) 81 ( 3
           81
           --
            ×
```

∴ H.C.F. of given numbers = 27.

Ex. 4. ***Reduce*** $\frac{391}{667}$ ***to lowest terms.***

Sol. H.C.F. of 391 and 667 is 23.

On dividing the numerator and denominator by 23, we get :

$$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}.$$

Ex. 5. ***Find the L.C.M. of*** $2^2 \times 3^3 \times 5 \times 7^2$, $2^3 \times 3^2 \times 5^2 \times 7^4$, $2 \times 3 \times 5^3 \times 7 \times 11$.

Sol. L.C.M. = Product of highest powers of 2, 3, 5, 7 and 11 = $2^3 \times 3^3 \times 5^3 \times 7^4 \times 11$

Ex. 6. ***Find the L.C.M. of 72, 108 and 2100.***

Sol. $72 = 2^3 \times 3^2$, $108 = 3^3 \times 2^2$, $2100 = 2^2 \times 5^2 \times 3 \times 7$.

∴ L.C.M. = $2^3 \times 3^3 \times 5^2 \times 7 = 37800$.

Ex. 7. ***Find the L.C.M. of 16, 24, 36 and 54.***

Sol.

2	16 –	24 –	36 –	54
2	8 –	12 –	18 –	27
2	4 –	6 –	9 –	27
3	2 –	3 –	9 –	27
3	2 –	1 –	3 –	9
	2 –	1 –	1 –	3

∴ L.C.M. = $2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 3 = 432$.

3. DECIMAL FRACTIONS

IMPORTANT FACTS AND FORMULAE

I. Decimal Fractions : Fractions in which denominators are powers of 10 are known as ***decimal fractions.***

Thus, $\frac{1}{10}$ = 1 tenth = .1; $\frac{1}{100}$ = 1 hundredth = .01;

$\frac{99}{100}$ = 99 hundredths = .99; $\frac{7}{1000}$ = 7 thousandths = .007, etc.

II. Conversion of a Decimal Into Vulgar Fraction : Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point. Now, remove the decimal point and reduce the fraction to its lowest terms.

Thus, $0.25 = \frac{25}{100} = \frac{1}{4}$; $2.008 = \frac{2008}{1000} = \frac{251}{125}$.

III. 1. Annexing zeros to the extreme right of a decimal fraction does not change its value.

Thus, 0.8 = 0.80 = 0.800, etc.

2. If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.

Thus, $\frac{1.84}{2.99} = \frac{184}{299} = \frac{8}{13}$; $\frac{.365}{.584} = \frac{365}{584} = \frac{5}{8}$

IV. Operations on Decimal Fractions :

1. **Addition and Subtraction of Decimal Fractions :** The given numbers are so placed under each other that the decimal points lie in one column. The numbers so arranged can now be added or subtracted in the usual way.

2. **Multiplication of a Decimal Fraction By a Power of 10 :** Shift the decimal point to the right by as many places as is the power of 10.

Thus, 5.9632 × 100 = 596.32; 0.073 × 10000 = 0.0730 × 10000 = 730.

3. **Multiplication of Decimal Fractions :** Multiply the given numbers considering them without the decimal point. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.

Suppose we have to find the product (.2 × .02 × .002).

Now, 2 × 2 × 2 = 8. Sum of decimal places = (1 + 2 + 3) = 6.

∴ .2 × .02 × .002 = .000008.

4. **Dividing a Decimal Fraction By a Counting Number :** Divide the given number without considering the decimal point, by the given counting number. Now, in the quotient, put the decimal point to give as many places of decimal as there are in the dividend.

Suppose we have to find the quotient (0.0204 ÷ 17). Now, 204 ÷ 17 = 12.

Dividend contains 4 places of decimal. So, 0.0204 ÷ 17 = 0.0012.

5. **Dividing a Decimal Fraction By a Decimal Fraction :** Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. Now, proceed as above.

Thus, $\frac{0.00066}{0.11} = \frac{0.00066 \times 100}{0.11 \times 100} = \frac{0.066}{11} = .006.$

V. **Comparison of Fractions :** Suppose some fractions are to be arranged in ascending or descending order of magnitude. Then, convert each one of the given fractions in the decimal form, and arrange them accordingly.

Suppose, we have to arrange the fractions $\frac{3}{5}, \frac{6}{7}$ and $\frac{7}{9}$ in descending order.

Now, $\frac{3}{5} = 0.6, \frac{6}{7} = 0.857, \frac{7}{9} = 0.777.....$

Since $0.857 > 0.777 ..... > 0.6$, so $\frac{6}{7} > \frac{7}{9} > \frac{3}{5}$.

VI. **Recurring Decimal :** If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a ***recurring decimal.***

In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set.

Thus, $\frac{1}{3} = 0.333 ..... = 0.\dot{3}; \frac{22}{7} = 3.142857142857 ..... = 3.\overline{142857}.$

Pure Recurring Decimal : A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal.

Converting a Pure Recurring Decimal Into Vulgar Fraction : Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

Thus, $0.\dot{5} = \frac{5}{9}; 0.\overline{53} = \frac{53}{99}; 0.\overline{067} = \frac{67}{999}$; etc.

Mixed Recurring Decimal : A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal.

e.g., $0.17333..... = 0.17\overline{3}$.

Converting a Mixed Recurring Decimal Into Vulgar Fraction : In the numerator, take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated. In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

Thus, $0.1\dot{6} = \frac{16-1}{90} = \frac{15}{90} = \frac{1}{6}; 0.22\overline{73} = \frac{2273-22}{9900} = \frac{2251}{9900}.$

VII. **Some Basic Formulae :**

1. $(a + b)(a - b) = (a^2 - b^2)$.
2. $(a + b)^2 = (a^2 + b^2 + 2ab)$.
3. $(a - b)^2 = (a^2 + b^2 - 2ab)$.
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$.
5. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
6. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$.
7. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$.
8. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

SOLVED EXAMPLES

Ex. 1. *Convert the following into vulgar fractions :*

(i) 0.75 *(ii)* 3.004 *(iii)* .0056.

Sol. (i) $0.75 = \frac{75}{100} = \frac{3}{4}$. (ii) $3.004 = \frac{3004}{1000} = \frac{751}{250}$. (iii) $.0056 = \frac{56}{10000} = \frac{7}{1250}$.

Ex. 2. *Arrange the fractions* $\frac{5}{8}, \frac{7}{12}, \frac{13}{16}, \frac{16}{29}$ *and* $\frac{3}{4}$ *in ascending order of magnitude.*

Sol. Converting each of the given fractions into decimal form, we get :

$\frac{5}{8} = 0.625$, $\frac{7}{12} = 0.5833$, $\frac{13}{16} = 0.8125$, $\frac{16}{29} = 0.5517$ and $\frac{3}{4} = 0.75$.

Now, $0.5517 < 0.5833 < 0.625 < 0.75 < 0.8125$.

$\therefore \quad \frac{16}{29} < \frac{7}{12} < \frac{5}{8} < \frac{3}{4} < \frac{13}{16}$.

Ex. 3. *Arrange the fractions* $\frac{3}{5}, \frac{4}{7}, \frac{8}{9}$ *and* $\frac{9}{11}$ *in their descending order.* **(R.B.I. 2003)**

Sol. Clearly, $\frac{3}{5} = 0.6$, $\frac{4}{7} = 0.571$, $\frac{8}{9} = 0.88$, $\frac{9}{11} = 0.818$.

Now, $0.88 > 0.818 > 0.6 > 0.571$.

$\therefore \quad \frac{8}{9} > \frac{9}{11} > \frac{3}{5} > \frac{4}{7}$.

Ex. 4. *Evaluate :* *(i)* 6202.5 + 620.25 + 62.025 + 6.2025 + 0.62025 **(L.I.C. 2003)**

(ii) 5.064 + 3.98 + .7036 + 7.6 + .3 + 2

Sol. (i)

$$\begin{array}{r} 6202.5 \\ 620.25 \\ 62.025 \\ 6.2025 \\ +\ 0.62025 \\ \hline 6891.59775 \end{array}$$

(ii)

$$\begin{array}{r} 5.064 \\ 3.98 \\ 0.7036 \\ 7.6 \\ 0.3 \\ +\ 2.0 \\ \hline 19.6476 \end{array}$$

Ex. 5. *Evaluate :* *(i)* 31.004 − 17.2386 *(ii)* 13 − 5.1967

Sol. (i)

$$\begin{array}{r} 31.0040 \\ -\ 17.2386 \\ \hline 13.7654 \end{array}$$

(ii)

$$\begin{array}{r} 13.0000 \\ -\ 5.1967 \\ \hline 7.8033 \end{array}$$

Ex. 6. *What value will replace the question mark in the following equations ?*

(i) 5172.49 + 378.352 + ? = 9318.678 **(B.S.R.B. 1998)**

(ii) ? − 7328.96 = 5169.38 **(B.S.R.B. 2003)**

Sol. (i) Let $5172.49 + 378.352 + x = 9318.678$.

Then, $x = 9318.678 - (5172.49 + 378.352) = 9318.678 - 5550.842 = 3767.836$.

(ii) Let $x - 7328.96 = 5169.38$. Then, $x = 5169.38 + 7328.96 = 12498.34$.

Ex. 7. *Find the products :* *(i)* 6.3204 × 100 *(ii)* .069 × 10000

Sol. (i) $6.3204 \times 100 = 632.04$. (ii) $.069 \times 10000 = .0690 \times 10000 = 690$.

4. SIMPLIFICATION

IMPORTANT CONCEPTS

I. **'BODMAS' Rule :** This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression.

Here, **'B'** stands for ***'Bracket'***, **'O'** for ***'of'***, **'D'** for ***'Division'***, **'M'** for ***'Multiplication'***, **'A'** for ***'Addition'*** and **'S'** for ***'Subtraction'***.

Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order (), {} and [].

After removing the brackets, we must use the following operations strictly in the order :

(*i*) of (*ii*) Division (*iii*) Multiplication (*iv*) Addition (*v*) Subtraction.

II. **Modulus of a Real Number :** Modulus of a real number a is defined as

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a, & \text{if } a < 0. \end{cases}$$

Thus, $|5| = 5$ and $|-5| = -(-5) = 5$.

III. **Virnaculum** (or Bar) : When an expression contains Virnaculum, before applying the 'BODMAS' rule, we simplify the expression under the Virnaculum.

SOLVED EXAMPLES

Ex. 1. *Simplify* : (*i*) *5005 − 5000 ÷ 10* (*ii*) *18800 ÷ 470 ÷ 20.*

Sol. (*i*) $5005 - 5000 \div 10 = 5005 - \dfrac{5000}{10} = 5005 - 500 = 4505.$

(*ii*) $18800 \div 470 \div 20 = \dfrac{18800}{470} \div 20 = 40 \div 20 = 2.$

Ex. 2. *Simplify* : $b - [b - (a + b) - \{b - (b - \overline{a - b})\} + 2a]$. **(Hotel Management, 2002)**

Sol. Given expression $= b - [b - (a + b) - \{b - (b - a + b)\} + 2a]$

$= b - [b - a - b - \{b - 2b + a\} + 2a]$

$= b - [-a - \{b - 2b + a + 2a\}]$

$= b - [-a - \{-b + 3a\}] = b - [-a + b - 3a]$

$= b - [-4a + b] = b + 4a - b = 4a.$

Ex. 3. *What value will replace the question mark in the following equation ?*

$$4\frac{1}{2} + 3\frac{1}{6} + ? + 2\frac{1}{3} = 13\frac{2}{5}.$$

Sol. Let $\dfrac{9}{2} + \dfrac{19}{6} + x + \dfrac{7}{3} = \dfrac{67}{5}$

Then, $x = \dfrac{67}{5} - \left(\dfrac{9}{2} + \dfrac{19}{6} + \dfrac{7}{3}\right) \Leftrightarrow x = \dfrac{67}{5} - \left(\dfrac{27 + 19 + 14}{6}\right) = \left(\dfrac{67}{5} - \dfrac{60}{6}\right)$

$\Leftrightarrow x = \left(\dfrac{67}{5} - 10\right) = \dfrac{17}{5} = 3\dfrac{2}{5}.$

Hence, missing fraction $= 3\dfrac{2}{5}.$

5. SQUARE ROOTS AND CUBE ROOTS

IMPORTANT FACTS AND FORMULAE

Square Root : If $x^2 = y$, we say that the square root of y is x and we write, $\sqrt{y} = x$.

Thus, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{196} = 14$.

Cube Root : The cube root of a given number x is the number whose cube is x. We denote the cube root of x by $\sqrt[3]{x}$.

Thus, $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$, $\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$ etc.

Note :

1. $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$
2. $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{x}}{\sqrt{y}} \times \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{xy}}{y}$.

SOLVED EXAMPLES

Ex. 1. ***Evaluate $\sqrt{6084}$ by factorization method.***

Sol. Method : Express the given number as the product of prime factors. Now, take the product of these prime factors choosing one out of every pair of the same primes. This product gives the square root of the given number.

2	6084
2	3042
3	1521
3	507
13	169
	13

Thus, resolving 6084 into prime factors, we get :

$6084 = 2^2 \times 3^2 \times 13^2$

$\therefore \quad \sqrt{6084} = (2 \times 3 \times 13) = 78.$

Ex. 2. ***Find the square root of 1471369.***

Sol. Explanation : In the given number, mark off the digits in pairs starting from the unit's digit. Each pair and the remaining one digit is called a period.

1	$\overline{1}\,\overline{47}\,\overline{13}\,\overline{69}$	(1213
	1	
22	47	
	44	
241	313	
	241	
2423	7269	
	7269	
	×	

Now, $1^2 = 1$. On subtracting, we get 0 as remainder.

Now, bring down the next period *i.e.*, 47.

Now, trial divisor is $1 \times 2 = 2$ and trial dividend is 47.

So, we take 22 as divisor and put 2 as quotient.

The remainder is 3.

Next, we bring down the next period which is 13.

Now, trial divisor is $12 \times 2 = 24$ and trial dividend is 313. So, we take 241 as dividend and 1 as quotient.

The remainder is 72.

Bring down the next period *i.e.*, 69.

Now, the trial divisor is $121 \times 2 = 242$ and the trial dividend is 7269. So, we take 3 as quotient and 2423 as divisor. The remainder is then zero.

Hence, $\sqrt{1471369} = 1213$.

6. AVERAGE

IMPORTANT FACTS AND FORMULAE

1. Average $= \left(\dfrac{\text{Sum of observations}}{\text{Number of observations}}\right)$
2. Suppose a man covers a certain distance at x kmph and an equal distance at y kmph. Then, the average speed during the whole journey is $\left(\dfrac{2xy}{x+y}\right)$ kmph.

SOLVED EXAMPLES

Ex. 1. ***Find the average of all prime numbers between 30 and 50.***

Sol. There are five prime numbers between 30 and 50.

They are 31, 37, 41, 43 and 47.

$\therefore$ Required average $= \left(\dfrac{31+37+41+43+47}{5}\right) = \dfrac{199}{5} = 39.8.$

Ex. 2. ***Find the average of first 40 natural numbers.***

Sol. Sum of first n natural numbers $= \dfrac{n(n+1)}{2}$

So, sum of first 40 natural numbers $= \dfrac{40 \times 41}{2} = 820.$

$\therefore$ Required average $= \dfrac{820}{40} = 20.5.$

Ex. 3. ***Find the average of first 20 multiples of 7.***

Sol. Required average $= \dfrac{7(1+2+3+\ldots\ldots+20)}{20} = \left(\dfrac{7 \times 20 \times 21}{20 \times 2}\right) = \left(\dfrac{147}{2}\right) = 73.5.$

Ex. 4. ***The average of four consecutive even numbers is 27. Find the largest of these numbers.***

Sol. Let the numbers be x, $x+2$, $x+4$ and $x+6$. Then,

$$\frac{x+(x+2)+(x+4)+(x+6)}{4} = 27 \Rightarrow \frac{4x+12}{4} = 27 \Rightarrow x+3 = 27 \Rightarrow x = 24.$$

$\therefore$ Largest number $= (x+6) = 24 + 6 = 30.$

Ex. 5. ***There are two sections A and B of a class, consisting of 36 and 44 students respectively. If the average weight of section A is 40 kg and that of section B is 35 kg, find the average weight of the whole class.***

Sol. Total weight of $(36 + 44)$ students $= (36 \times 40 + 44 \times 35)$ kg $= 2980$ kg.

$\therefore$ Average weight of the whole class $= \left(\dfrac{2980}{80}\right)$ kg $= 37.25$ kg.

7. PROBLEMS ON NUMBERS

In this section, questions involving a set of numbers are put in the form of a puzzle. You have to analyse the given conditions, assume the unknown numbers and form equations accordingly, which on solving yield the unknown numbers.

SOLVED EXAMPLES

Ex. 1. ***A number is as much greater than 36 as is less than 86. Find the number.***

Sol. Let the number be x. Then, $x - 36 = 86 - x \Leftrightarrow 2x = 86 + 36 = 122 \Leftrightarrow x = 61$.

Hence, the required number is 61.

Ex. 2. ***Find a number such that when 15 is subtracted from 7 times the number, the result is 10 more than twice the number.*** (Hotel Management, 2002)

Sol. Let the number be x. Then, $7x - 15 = 2x + 10 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5$.

Hence, the required number is 5.

Ex. 3. ***The sum of a rational number and its reciprocal is $\frac{13}{6}$. Find the number.*** (S S.C. 2000)

Sol. Let the number be x.

$$\text{Then, } x + \frac{1}{x} = \frac{13}{6} \Leftrightarrow \frac{x^2+1}{x} = \frac{13}{6} \Leftrightarrow 6x^2 - 13x + 6 = 0$$

$$\Leftrightarrow 6x^2 - 9x - 4x + 6 = 0 \Leftrightarrow (3x - 2)(2x - 3) = 0$$

$$\Leftrightarrow x = \frac{2}{3} \text{ or } x = \frac{3}{2}.$$

Hence, the required number is $\frac{2}{3}$ or $\frac{3}{2}$.

Ex. 4. ***The sum of two numbers is 184. If one-third of the one exceeds one-seventh of the other by 8, find the smaller number.***

Sol. Let the numbers be x and $(184 - x)$. Then,

$$\frac{x}{3} - \frac{(184 - x)}{7} = 8 \Leftrightarrow 7x - 3(184 - x) = 168 \Leftrightarrow 10x = 720 \Leftrightarrow x = 72.$$

So, the numbers are 72 and 112. Hence, smaller number = 72.

Ex. 5. ***The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers.***

Sol. Let the numbers be x and y. Then.

$$x - y = 11 \quad ...(i) \qquad \text{and} \quad \frac{1}{5}(x + y) = 9 \Rightarrow x + y = 45 \quad ...(ii)$$

Adding (*i*) and (*ii*), we get : $2x = 56$ or $x = 28$. Putting $x = 28$ in (*i*), we get : $y = 17$.

Hence, the numbers are 28 and 17.

Ex. 6. ***If the sum of two numbers is 42 and their product is 437, then find the absolute difference between the numbers.*** (S.S.C. 2003)

Sol. Let the numbers be x and y. Then, $x + y = 42$ and $xy = 437$.

$$x - y = \sqrt{(x + y)^2 - 4xy} = \sqrt{(42)^2 - 4 \times 437} = \sqrt{1764 - 1748} = \sqrt{16} = 4.$$

$\therefore$ Required difference = 4.

8. PROBLEMS ON AGES

SOLVED EXAMPLES

Ex. 1. ***Rajeev's age after 15 years will be 5 times his age 5 years back. What is the present age of Rajeev ?*** (Hotel Management, 2002)

Sol. Let Rajeev's present age be x years. Then,

Rajeev's age after 15 years = $(x + 15)$ years.

Rajeev's age 5 years back = $(x - 5)$ years.

$\therefore \quad x + 15 = 5(x - 5) \Leftrightarrow x + 15 = 5x - 25 \Leftrightarrow 4x = 40 \Leftrightarrow x = 10.$

Hence, Rajeev's present age = 10 years.

Ex. 2. ***The ages of two persons differ by 16 years. If 6 years ago, the elder one be 3 times as old as the younger one, find their present ages.*** (A.A.O. Exam, 2003)

Sol. Let the age of the younger person be x years.

Then, age of the elder person = $(x + 16)$ years.

$\therefore \quad 3(x - 6) = (x + 16 - 6) \Leftrightarrow 3x - 18 = x + 10 \Leftrightarrow 2x = 28 \Leftrightarrow x = 14.$

Hence, their present ages are 14 years and 30 years.

Ex. 3. ***The product of the ages of Ankit and Nikita is 240. If twice the age of Nikita is more than Ankit's age by 4 years, what is Nikita's age ?*** (S.B.I.P.O. 1999)

Sol. Let Ankit's age be x years. Then, Nikita's age = $\dfrac{240}{x}$ years.

$$\therefore \quad 2 \times \frac{240}{x} - x = 4 \Leftrightarrow 480 - x^2 = 4x \Leftrightarrow x^2 + 4x - 480 = 0$$

$$\Leftrightarrow (x + 24)(x - 20) = 0 \Leftrightarrow x = 20.$$

Hence, Nikita's age = $\left(\dfrac{240}{20}\right)$ years = 12 years.

Ex. 4. ***The present age of a father is 3 years more than three times the age of his son. Three years hence, father's age will be 10 years more than twice the age of the son. Find the present age of the father.*** (S.S.C. 2003)

Sol. Let the son's present age be x years. Then, father's present age = $(3x + 3)$ years.

$\therefore \quad (3x + 3 + 3) = 2(x + 3) + 10 \Leftrightarrow 3x + 6 = 2x + 16 \Leftrightarrow x = 10.$

Hence, father's present age = $(3x + 3) = (3 \times 10 + 3)$ years = 33 years.

Ex. 5. ***Rohit was 4 times as old as his son 8 years ago. After 8 years, Rohit will be twice as old as his son. What are their present ages ?***

Sol. Let son's age 8 years ago be x years. Then, Rohit's age 8 years ago = $4x$ years.

Son's age after 8 years = $(x + 8) + 8 = (x + 16)$ years.

Rohit's age after 8 years = $(4x + 8) + 8 = (4x + 16)$ years.

$\therefore \quad 2(x + 16) = 4x + 16 \Leftrightarrow 2x = 16 \Leftrightarrow x = 8.$

Hence, son's present age = $(x + 8)$ = 16 years.

Rohit's present age = $(4x + 8)$ = 40 years.

Ex. 6. ***One year ago, the ratio of Gaurav's and Sachin's age was 6 : 7 respectively. Four years hence, this ratio would become 7 : 8. How old is Sachin ?*** (NABARD, 2002)

Sol. Let Gaurav's and Sachin's ages one year ago be $6x$ and $7x$ years respectively. Then,

Gaurav's age 4 years hence = $(6x + 1) + 4 = (6x + 5)$ years.

Sachin's age 4 years hence = $(7x + 1) + 4 = (7x + 5)$ years.

$$\therefore \quad \frac{6x+5}{7x+5} = \frac{7}{8} \Leftrightarrow 8(6x+5) = 7(7x+5) \Leftrightarrow 48x + 40 = 49x + 35 \Leftrightarrow x = 5.$$

Hence, Sachin's present age = $(7x + 1) = 36$ years.

Ex. 7. ***Abhay's age after six years will be three-seventh of his father's age. Ten years ago, the ratio of their ages was 1 : 5. What is Abhay's father's age at present ?***

Sol. Let the ages of Abhay and his father 10 years ago be x and $5x$ years respectively. Then,

Abhay's age after 6 years = $(x + 10) + 6 = (x + 16)$ years.

Father's age after 6 years = $(5x + 10) + 6 = (5x + 16)$ years.

$$\therefore \quad (x+16) = \frac{3}{7}(5x+16) \Leftrightarrow 7(x+16) = 3(5x+16) \Leftrightarrow 7x + 112 = 15x + 48$$

$$\Leftrightarrow 8x = 64 \Leftrightarrow x = 8.$$

Hence, Abhay's father's present age = $(5x + 10) = 50$ years.

9. SURDS AND INDICES

IMPORTANT FACTS AND FORMULAE

1. LAWS OF INDICES :

(i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$ (iii) $(a^m)^n = a^{mn}$

(iv) $(ab)^n = a^n b^n$ (v) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ (vi) $a^0 = 1$

2. SURDS : *Let a be a rational number and n be a positive integer such that* $a^{\frac{1}{n}} = \sqrt[n]{a}$ *is irrational. Then,* $\sqrt[n]{a}$ *is called a surd of order n.*

3. LAWS OF SURDS :

(i) $\sqrt[n]{a} = a^{\frac{1}{n}}$ (ii) $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$ (iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iv) $(\sqrt[n]{a})^n = a$ (v) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$. (vi) $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$.

SOLVED EXAMPLES

Ex. 1. *Simplify :* (i) $(27)^{\frac{2}{3}}$ (ii) $(1024)^{-\frac{4}{5}}$ (iii) $\left(\frac{8}{125}\right)^{-\frac{4}{3}}$.

Sol. (i) $(27)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^{\left(3 \times \frac{2}{3}\right)} = 3^2 = 9.$

(ii) $(1024)^{-\frac{4}{5}} = (4^5)^{-\frac{4}{5}} = 4^{\left\{5 \times \frac{(-4)}{5}\right\}} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}.$

(iii) $\left(\frac{8}{125}\right)^{-\frac{4}{3}} = \left\{\left(\frac{2}{5}\right)^3\right\}^{-\frac{4}{3}} = \left(\frac{2}{5}\right)^{\left\{3 \times \frac{(-4)}{3}\right\}} = \left(\frac{2}{5}\right)^{-4} = \left(\frac{5}{2}\right)^4 = \frac{5^4}{2^4} = \frac{625}{16}.$

Ex. 2. *Evaluate :* (i) $(.00032)^{\frac{3}{5}}$ (ii) $(256)^{0.16} \times (16)^{0.18}$.

Sol. (i) $(0.00032)^{\frac{3}{5}} = \left(\frac{32}{100000}\right)^{\frac{3}{5}} = \left(\frac{2^5}{10^5}\right)^{\frac{3}{5}} = \left\{\left(\frac{2}{10}\right)^5\right\}^{\frac{3}{5}} = \left(\frac{1}{5}\right)^{\left(5 \times \frac{3}{5}\right)} = \left(\frac{1}{5}\right)^3 = \frac{1}{125}.$

(ii) $(256)^{0.16} \times (16)^{0.18} = \{(16)^2\}^{0.16} \times (16)^{0.18} = (16)^{(2 \times 0.16)} \times (16)^{0.18}$

$= (16)^{0.32} \times (16)^{0.18} = (16)^{(0.32 + 0.18)} = (16)^{0.5} = (16)^{\frac{1}{2}} = 4.$

10. PERCENTAGE

IMPORTANT FACTS AND FORMULAE

I. Concept of Percentage : By a certain *percent*, we mean that many hundredths. Thus, x percent means x hundredths, written as $x\%$.

To express x% as a fraction : We have, $x\% = \frac{x}{100}$.

Thus, $20\% = \frac{20}{100} = \frac{1}{5}$; $48\% = \frac{48}{100} = \frac{12}{25}$, etc.

To express $\frac{a}{b}$ as a percent : We have, $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$.

Thus, $\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = 25\%$; $0.6 = \frac{6}{10} = \frac{3}{5} = \left(\frac{3}{5} \times 100\right)\% = 60\%$.

II. If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is

$$\left[\frac{R}{(100+R)} \times 100\right]\%$$

If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is

$$\left[\frac{R}{(100-R)} \times 100\right]\%$$

III. Results on Population : Let the population of a town be P now and suppose it increases at the rate of R% per annum, then :

1. Population after n years $= P\left(1 + \frac{R}{100}\right)^n$

2. Population n years ago $= \frac{P}{\left(1 + \frac{R}{100}\right)^n}$.

IV. Results on Depreciation : Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then :

1. Value of the machine after n years $= P\left(1 - \frac{R}{100}\right)^n$

2. Value of the machine n years ago $= \frac{P}{\left(1 - \frac{R}{100}\right)^n}$.

V. If A is R% more than B, then B is less than A by

$$\left[\frac{R}{(100+R)} \times 100\right]\%.$$

If A is R% less than B, then B is more than A by

$$\left[\frac{R}{(100-R)} \times 100\right]\%.$$

SOLVED EXAMPLES

Ex. 1. ***Express each of the following as a fraction :***

(i) 56% *(ii) 4%* *(iii) 0.6%* *(iv) 0.08%*

Sol. (i) $56\% = \frac{56}{100} = \frac{14}{25}$. (ii) $4\% = \frac{4}{100} = \frac{1}{25}$.

(iii) $0.6\% = \frac{0.6}{100} = \frac{6}{1000} = \frac{3}{500}$. (iv) $0.08\% = \frac{0.08}{100} = \frac{8}{10000} = \frac{1}{1250}$.

Ex. 2. ***Express each of the following as a decimal :***

(i) 6% *(ii) 28%* *(iii) 0.2%* *(iv) 0.04%*

Sol. (i) $6\% = \frac{6}{100} = 0.06$. (ii) $28\% = \frac{28}{100} = 0.28$.

(iii) $0.2\% = \frac{0.2}{100} = 0.002$. (iv) $0.04\% = \frac{0.04}{100} = 0.0004$.

Ex. 3. ***Express each of the following as rate percent :***

(i) $\frac{23}{36}$ *(ii)* $6\frac{3}{4}$ *(iii) 0.004*

Sol. (i) $\frac{23}{36} = \left(\frac{23}{36} \times 100\right)\% = \left(\frac{575}{9}\right)\% = 63\frac{8}{9}\%$.

(ii) $0.004 = \frac{4}{1000} = \left(\frac{4}{1000} \times 100\right)\% = 0.4\%$.

(iii) $6\frac{3}{4} = \frac{27}{4} = \left(\frac{27}{4} \times 100\right)\% = 675\%$.

Ex. 4. ***Evaluate :***

(i) 28% of 450 + 45% of 280 (Bank P.O. 2003)

(ii) $16\frac{2}{3}\%$ *of 600 gm –* $33\frac{1}{3}\%$ *of 180 gm* (R.R.B. 1998)

Sol. (i) 28% of $450 + 45\%$ of $280 = \left(\frac{28}{100} \times 450 + \frac{45}{100} \times 280\right) = (126 + 126) = 252$.

(ii) $16\frac{2}{3}\%$ of 600 gm – $33\frac{1}{3}\%$ of 180 gm

$= \left[\left(\frac{50}{3} \times \frac{1}{100} \times 600\right) - \left(\frac{100}{3} \times \frac{1}{100} \times 180\right)\right]$ gm $= (100 - 60)$ gm $= 40$ gm.

Ex. 5. *(i) 2 is what percent of 50 ?* (S.S.C. 2000)

(ii) $\frac{1}{2}$ *is what percent of* $\frac{1}{3}$ *?* (S.S.C. 2002)

(iii) What percent of 7 is 84 ?

(iv) What percent of 2 metric tonnes is 40 quintals ?

(v) What percent of 6.5 litres is 130 ml ?

Sol. (i) Required percentage $= \left(\frac{2}{50} \times 100\right)\% = 4\%$.

(ii) Required percentage $= \left(\frac{1}{2} \times \frac{3}{1} \times 100\right)\% = 150\%$.

(iii) Required percentage $= \left(\frac{84}{7} \times 100\right)\% = 1200\%$.

(*iv*) 1 metric tonne = 10 quintals.

$\therefore$ Required percentage $= \left(\dfrac{40}{2 \times 10} \times 100\right)\% = 200\%.$

(*v*) Required percentage $= \left(\dfrac{130}{6.5 \times 1000} \times 100\right)\% = 2\%.$

Ex. 6. ***Find the missing figures :***

(i) ?% of 25 = 2.125 ***(ii) 9% of ? = 63*** ***(iii) 0.25% of ? = 0.04***

Sol. (*i*) Let $x\%$ of 25 = 2.125. Then, $\dfrac{x}{100} \times 25 = 2.125 \Leftrightarrow x = (2.125 \times 4) = 8.5.$

(*ii*) Let 9% of x = 6.3. Then, $\dfrac{9}{100} x = 6.3 \Leftrightarrow x = \left(\dfrac{6.3 \times 100}{9}\right) = 70.$

(*iii*) Let 0.25% of x = 0.04. Then, $\dfrac{0.25}{100} x = 0.04 \Leftrightarrow x = \left(\dfrac{0.04 \times 100}{0.25}\right) = 16.$

Ex. 7. ***Which is greatest in $16\frac{2}{3}\%$, $\frac{2}{15}$ and 0.17 ?***

Sol. $16\frac{2}{3}\% = \left(\dfrac{50}{3} \times \dfrac{1}{100}\right) = \dfrac{1}{6} = 0.166,\ \dfrac{2}{15} = 0.133.$ Clearly, 0.17 is the greatest.

Ex. 8. ***If the sales tax be reduced from $3\frac{1}{2}\%$ to $3\frac{1}{3}\%$, then what difference does it make to a person who purchases an article with marked price of Rs. 8400 ?*** **(S.S.C. 2002)**

Sol. Required difference $= \left(3\frac{1}{2}\% \text{ of Rs. } 8400\right) - \left(3\frac{1}{3}\% \text{ of Rs. } 8400\right)$

$= \left(\dfrac{7}{2} - \dfrac{10}{3}\right)\% \text{ of Rs. } 8400 = \dfrac{1}{6}\% \text{ of Rs. } 8400$

$= \text{Rs.} \left(\dfrac{1}{6} \times \dfrac{1}{100} \times 8400\right) = \text{Rs. } 14.$

Ex. 9. ***An inspector rejects 0.08% of the meters as defective. How many will he examine to reject 2 ?*** **(M.A.T. 2000)**

Sol. Let the number of meters to be examined be x.

Then, 0.08% of $x = 2 \Leftrightarrow \left(\dfrac{8}{100} \times \dfrac{1}{100} \times x\right) = 2 \Leftrightarrow x = \left(\dfrac{2 \times 100 \times 100}{8}\right) = 2500.$

Ex. 10. ***Sixty-five percent of a number is 21 less than four-fifth of that number. What is the number ?***

Sol. Let the number be x.

Then, $\dfrac{4}{5}x - (65\% \text{ of } x) = 21 \Leftrightarrow \dfrac{4}{5}x - \dfrac{65}{100}x = 21 \Leftrightarrow 15x = 2100 \Leftrightarrow x = 140.$

Ex. 11. ***Difference of two numbers is 1660. If 7.5% of one number is 12.5% of the other number, find the two numbers.***

Sol. Let the numbers be x and y. Then, 7.5% of x = 12.5% of $y \Leftrightarrow x = \dfrac{125}{75} y = \dfrac{5}{3} y.$

Now, $x - y = 1660 \Rightarrow \dfrac{5}{3}y - y = 1660 \Rightarrow \dfrac{2}{3}y = 1660 \Rightarrow y = \left(\dfrac{1660 \times 3}{2}\right) = 2490.$

$\therefore$ One number = 2490, Second number $= \dfrac{5}{3}y = 4150.$

11. PROFIT AND LOSS

IMPORTANT FACTS

Cost Price : The price at which an article is purchased, is called its ***cost price***, abbreviated as *C.P.*

Selling Price : The price at which an article is sold, is called its ***selling price***, abbreviated as *S.P.*

Profit or Gain : If S.P. is greater than C.P., the seller is said to have a ***profit*** or ***gain***.

Loss : If S.P. is less than C.P., the seller is said to have incurred a ***loss***.

FORMULAE

1. Gain = (S.P.) – (C.P.)
2. Loss = (C.P.) – (S.P.)
3. Loss or gain is always reckoned on C.P.
4. $\text{Gain\%} = \left(\frac{\text{Gain} \times 100}{\text{C.P.}}\right)$
5. $\text{Loss\%} = \left(\frac{\text{Loss} \times 100}{\text{C.P.}}\right)$
6. $\text{S.P.} = \frac{(100 + \text{Gain\%})}{100} \times \text{C.P.}$
7. $\text{S.P.} = \frac{(100 - \text{Loss\%})}{100} \times \text{C.P.}$
8. $\text{C.P.} = \frac{100}{(100 + \text{Gain\%})} \times \text{S.P.}$
9. $\text{C.P.} = \frac{100}{(100 - \text{Loss\%})} \times \text{S.P.}$
10. If an article is sold at a gain of say, 35%, then S.P. = 135% of C.P.
11. If an article is sold at a loss of say, 35%, then S.P. = 65% of C.P.
12. When a person sells two similar items, one at a gain of say, $x\%$, and the other at a loss of $x\%$, then the seller always incurs a loss given by :

$$\text{Loss\%} = \left(\frac{\text{Common Loss and Gain\%}}{10}\right)^2 = \left(\frac{x}{10}\right)^2$$

13. If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain\%} = \left[\frac{\text{Error}}{(\text{True Value}) - (\text{Error})} \times 100\right]\%.$$

SOLVED EXAMPLES

Ex. 1. ***A man buys an article for Rs. 27.50 and sells it for Rs. 28.60. Find his gain percent.***

Sol. C.P. = Rs. 27.50, S.P. = Rs. 28.60.

So, Gain = Rs. (28.60 – 27.50) = Rs. 1.10.

$\therefore \quad \text{Gain\%} = \left(\frac{1.10}{27.50} \times 100\right)\% = 4\%.$